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Introducing bilateral exchange rates in global CGE models

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Contents

Introducing bilateral exchange rates in global CGE models[*]

Abstract

Nominal exchange rates play a key role in adjusting international relative prices and in balancing external accounts. But most global CGE models do not identify exchange rates, since they are mainly designed to analyse changes in quantities, which are ruled by changes in relative prices. Adding nominal exchange rates to such models allows individual countries to use their own currencies and enables nominal exchange rates to play a central role in adjusting relative prices between countries toward equilibrium. Despite having no impact on real results, adding nominal exchange rates makes it easier to explain how international relative prices adjust to balance external accounts for all countries.

This paper discusses the role of national currencies and nominal exchange rates in CGE models. It uses a prototype model to demonstrate the theoretical properties of a multi-country CGE model with currencies and exchange rates. It also uses a global CGE model using the GTAP data to show what is required to introduce bilateral nominal exchange rates into a conventional global CGE model and how this facilitates the implementation and the interpretation of alternative price adjustment mechanisms in the new model.

Introduction

The nominal exchange rate (NER) is a standard component of single-country computable equilibrium (CGE) models, but absent in most global CGE models. Why do some CGE models include NERs as a variable and some not, and what role, if any, should NERs play in CGE models? [1] These questions and many others regarding NERs and CGE models have not been given clear explanations in the literature.

For many people, the reason for the absence of NERs in global CGE models seems to be obvious, because these are real economy models and any price variable in such models

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1 Robinson (2003) raises a similar question, 'Does a CGE model have a meaningful exchange rate?' In his presentation, however, it is difficult to distinguish whether the author is referring to NERs or real exchange rates.

represents only a relative price. Therefore, NERs, if introduced as a relative price not a financial asset, should have no real impact on model results. For example, McDougall, et al. (2012, p. 7) argue in the context of the GTAP model:

> The choice of not having a nominal exchange rate in the model is associated with the underlying framework. The model does not discuss absolute price levels. In other words, all prices are relative to a numeraire so that changing only the nominal values will not be effective. Moreover, the demand functions are homogeneous of degree zero in prices which means that following a nominal exchange rate shock, there will be no change in quantity demanded.

It is true that the NER can be used as a numeraire in single country models and, therefore, should have no real impact on model results. However, this is not a sufficient reason to exclude NERs from global CGE models. In the real world, the domestic prices of all countries are denominated in their own currencies. NERs are used to convert national currency prices between countries to facilitate international trade and investment. In practice, the domestic prices of a country tend to be stable and NERs adjust to required changes in relative prices with other countries and to balance its external account.

Single-country models are capable of capturing such real world phenomena, because they include as a closure option to choose a domestic price as the numeraire and set the NER adjustable. Making this closure option available in global CGE models would mean that countries in the global model can be treated as truly independent national economies with their own currencies and nominal prices. In addition, expressing results in domestic currency terms, instead of a common foreign currency, makes their interpretation more accessible to readers, who are familiar with the units of measurement in their own currency. Moreover, the introduction of national currencies and NERs in a global CGE model is an important step toward a more sophisticated financial CGE model, in which the national currency is also a key ingredient of financial assets. Such a model could incorporate new monetary policy instruments with new agents and behaviours, which could bring about real impacts in the model results.

Few attempts have been made so far to incorporate NERs into global CGE models (see McDonald et al. (2007) and Lemelin et al. (2013)). However, NERs in these models are defined as effective rates, that is, on a one country versus the rest of the world (RoW) basis. This is inconsistent with the bilateral nature of exchange rates. This paper offers a new approach by explicitly introducing national currencies into a global CGE model so that NERs can be defined naturally on a bilateral basis as the ratios of two national currencies. As the new model is based on a fully bilateral database, the price systems of individual countries can be denominated in their own currencies. This allows the nominal price level of a country to be independently determined by a country specific numeraire. Either the currency or a price, or a national price index, could be chosen as the national numeraire. The replacement of the single global numeraire with replaced by multiple numeraires does not change the relative prices of the original model, so that the homogeneity nature of the model remains intact.

The remainder of this paper is organised as follows. The first section discusses the nature of the currencies introduced in the model, why they should have no real effect on quantity results, and how the approach differs from previous ones. In the second section, prototype models are used to illustrate numerically the role of national currencies and exchange rates in a multi-country CGE model. Introducing bilateral exchange rates requires the model database to describe all bilateral relationships between countries. The third section uses the GTAP database to illustrate how to bilateralise this database and how to modify the equation system to incorporate currencies and NERs. Test simulations show how exchange rates facilitate international price adjustments and confirm that the usual properties of the original CGE model remain intact. The paper concludes with some remarks on how this paper can be used to extend a global CGE model to include financial assets.

Nominal exchange rates and CGE models

This section is concerned with some conceptual issues on currencies and their exchange rates and also their role in CGE models in general, and global models in particular.

Currencies and exchange rates: some conceptual issues[2]

A nominal exchange rate (NER) is a ratio of two national currencies (NC). For example, the NER for home country *s* can be defined as the ratio of foreign country *r*'s currency to the home currency as follows,

$$(1) \qquad NER_{(r,s)} = \frac{NC_{(s)}}{NC_{(r)}}$$

It defines a country's NER as the units of the home currency that are required to purchase one unit of a foreign currency. The value of the domestic currency is, therefore, the inverse of this definition: a rise (fall) in $NC_{(s)}$, relative to $NC_{(r)}$, implies a depreciation (appreciation) of the home currency, which results in a rise (fall) in its bilateral foreign exchange rates, $NER_{(r,s)}$. This is equivalent to an appreciation (depreciation) of the values of all foreign currencies, relative to the home currency.

The above definition reveals three important features of the NER. Firstly, the NER should be bilateral, implying $NER_{(r,s)} = 1/NER_{(r,s)}$. Secondly, bilateral NERs are independent of the value, or the units, of individual currencies: only their relative values matter. This is because, in a multi-country world, the value of one currency is determined endogenously by the values of other currencies. As a result, the value of one currency can be normalised as unity so that the values of all other currencies is measured against it. Thirdly, the NER is a conversion factor, whose size is determined by the values of the national currencies involved. Therefore,

2 This section is based on the standard theory of exchange rates in open-economy macroeconomics. A good reference, consistent with the approach followed here, is found in Part 3 of Krugman, et al. (2015).

understanding the nature of national currencies is the key to our understanding of the role of NER in CGE models.

A national currency plays multiple functions in an economy. First of all, it is a medium of exchange: a national currency denominates the values of all products in an economy so that they can all be measured in the same unit in transactions. However, this role, as a medium of exchange, also gives national currency a value: it is the purchasing power of a basket of goods and services produced in the country. As a result, a national currency can be a store of value and can therefore be traded as a *financial asset* in foreign exchange markets. Its price is determined by the interactions of its supply and demand.

This paper does not discuss national currencies as a financial asset. Instead, it focuses exclusively on its role as a medium of exchange. This role is consistent with a conventional CGE model, without financial assets. In this context, a NER acts only as a conversion factor, used to convert national prices from one currency to another.

The value of one unit of a national currency is determined by its purchasing power in terms of a basket goods and services. However, the prices of the same goods and services, produced in different countries, are not directly comparable because they are denominated in different currencies. The role of a NER is to convert the prices of goods and services, denominated in different currencies, into a common unit, so that their true costs become comparable.

Due to the law of one price (LOP), the purchasing power of a domestic currency for a specific basket of goods and services should be equal to the purchasing power of a foreign currency for the same goods and services, when it is converted into the domestic currency price using a NER.

$$(2) \qquad NER_{(r,s)}\, P_{(r)} = P_{(s)}$$

where $P_{(r)}$ ($P_{(s)}$) is the price of a basket of foreign (home) goods and services, denominated in the foreign (home) currency. As a result, the NER can be defined as:

$$(3) \qquad NER_{(r,s)} = \frac{P_{(s)}}{P_{(r)}}$$

This is the so-called purchasing power parity (PPP) theory of exchange rates, which states that, in the long run, the exchange rate between domestic and foreign currencies tends toward the ratio of the domestic price level to the foreign price level. This relationship is based on a common basket of goods and services. As the exchange rate is normally related to the price of a basket of goods and services for the country as a whole, a national price index, such as the GDP deflator, is often used in the above definition. However, the goods and services used by different countries to compile their national price indexes differ usually. Therefore, the above equation typically does not hold when national price indexes are used. As a result, this NER definition is often expressed in its relative form as the ratio of *changes* in two national prices ($NP_{(r)}$),

(4) $ner_{(r,s)} = np_{(s)} - np_{(r)}$

where the lower cases refer to percentage changes. Equation 4 shows that the purchasing power of a currency is fundamentally determined by the price of its national output. As the NER is a ratio of two currencies (equation 1), the changes in the value of a national currency can be related to the changes in the underlying national price level.

Defining the NER as the relative purchasing power of a currency leads to another important concept, the real exchange rate (RER). The RER is an indicator for a country's IRP, which can be defined as the ratio of the foreign price level to the domestic price level, where the foreign price is converted into the domestic currency price using a NER. In the relative form, the RER is defined as:

(5) $rer_{(r,s)} = ner_{(r,s)} + np_{(r)} - np_{(s)} = (nc_{(r)} - nc_{(s)}) + (np_{(r)} - np_{(s)})$

The IRP of a country should always be adjustable, because it is a key price signal that guides an economy toward external equilibrium. Changes in RERs reflect the IRP adjustments that are required to balance a country's external account. If the value of a country's currency is fixed, the required adjustment in its IRP has to be made through flexible domestic prices. On the other hand, if a country's domestic prices are not fully flexible, the required changes in its IRP have to be made through a flexible currency and its bilateral NERs.

Equation 5 shows that the role of a country's bilateral NERs is to provide an alternative price adjustment mechanism. When the domestic price of a country is not fully responsive, the value of its currency can adjust relative to that of foreign currencies so that the required adjustment in the country's IRP can still be made. This is also the role that national currencies and NERs should play in CGE models.

The role of nominal exchange rates in CGE models

The solution to a CGE model is a set of relative prices that clear all markets. A necessary condition for such a solution is that all prices are able to adjust in response to changes in supply and demand. In an open-economy model, two types of relative prices can be identified: domestic relative prices and IRP. The former refer to relative prices between industries, factors or final users, while the latter refers to the general price levels between countries. Adjustments in domestic relative prices are required to clear all domestic markets. However, this is not enough for a general equilibrium solution in an open economy: a country's external accounts might not balance if some of its prices relative to the rest of the world cannot adjust.

For a country's external account to balance, its international relative price (IRP), often measured as the RER, must be flexible. The ability of a country's IRP to adjust depends on two factors: a flexible domestic price system or a flexible NER. If all prices are fully flexible, a country may not need a flexible NER to reach external balance. However, if some domestic prices are not fully adjustable, a flexible NER becomes necessary for external account balance.s

Understanding the role of NERs in open economy provides a key to explaining why single country CGE models need a NER, but global models do not. All single-country open-economy models include the NER as a variable. This is because, in single-country models, the world prices that the country faces are assumed to be given. If a domestic price is fixed as a numeraire, the domestic price level is constrained and cannot be adjusted, relative to foreign prices, to balance its external account. Therefore, a typical closure option for such a model is either fixing the NER, with all domestic prices set as endogenous, or fixing a domestic price or price index, such as the CPI, with the NER set as endogenous. This numeraire swap between NER and CPI reflects the importance of the NER in adjusting a country's IRP for external balance.

The absence of the RoW as an independent economy makes the NER a necessary component of the single-country model. If the RoW were modelled as an independent economy in a two-country model, the NER might not be needed. This is because, in such a model, prices and income in the RoW economy would be able to adjust, and the external accounts of the two economies would balance automatically. However, if the RoW is another economy, this model is no longer a single-country model, but a simple version of a global model.

In a global CGE model, if all prices are fully flexible internationally, NERs become redundant because the required adjustments in IRPs for external balance can be made through domestic price changes directly. *In the real world, however, not all domestic prices in a country are fully flexible. This is why NERs play an active role in facilitating trade and investment across countries in the world economy*. Excluding NERs in a global model is based on the implicit assumption that NERs are fixed, which is inconsistent with what occurs in the real world.[3]

Perhaps due to this implicit assumption, the importance of NERs in global CGE models has long been overlooked or dismissed as unnecessary in the literature. So far, only few CGE modellers have attempted to introduce NERs into global models. For example, McDonald et al. (2007)[4] develop a global CGE model, GLOBE, which accounts explicitly for NERs. The exchange rate in this model is defined as an *effective exchange rate* (EER), the price of an implicit "international currency" in terms of each country's domestic currency. Lemelin, et al. (2013) also follow this approach in their global model, PEP-w-1. The definition of NERs as EERs is inconsistent with the bilateral nature of inter-country relationship in these global models. The working of these EERs relies heavily on an imaginary "world market" with an implicit "international currency". The original bilateral trade between countries has to go through a third party, the world market, which makes the interpretation of international price adjustments unnecessarily complicated.

3 Except in cases, now rare, when some NERs are pegged bilaterally.

4 A new version of this model can be found in McDonald and Thierfelder (2016).

Naturally, in a global model, NERs should be defined on a bilateral basis as ratios of national currencies. This definition is in line with the bilateral nature of the multi-country global model structure. If national currencies are introduced as a variable, all bilateral exchange rates can be readily defined as the ratios of corresponding national currencies. The introduction of national currencies allows each country's domestic prices to be denominated in its own currency. Trade transactions and income transfers can be conducted using bilateral exchange rates. In this context, the domestic-currency price of an export can be treated naturally as the foreign price by its importing countries and converted into its domestic price using a bilateral NER. There is no need for an arbitrary international currency or world market to facilitate the trade and income transfer.

The introduction of national currencies does not alter the underlying IRPs and, therefore, no real change is expected in model results. However, the introduction of national currencies changes the units of measurement in the nominal prices of a global model. It allows each country to have an independent price system, denominated in its own national currency. As a result, the single global numeraire is no longer needed and can be replaced by multiple national numeraires. This implies that each country can choose one of its own domestic prices as a national numeraire and set its currency flexible so that the country's IRPs can still be adjusted by its bilateral NERs. The national numeraire is a useful feature that makes it possible to implement diverse economic environments at the national or regional level in a global model context.

However, introducing bilateral exchange rates and national currencies requires full bilateral price transmissibility in the model's database. That is, any nominal flow in the database must be transmissible from one country's price to another's by using a bilateral exchange rate. This requirement is not always satisfied in global model databases, such as GTAP.

This paper offers some simple methods to resolve these data problems. Before addressing these problems inherent to applied models, a prototype model is used in the next section to illustrate numerically the roles that national currencies and NERs play in a global CGE model.

Numerical examples with a prototype model

In this section, a prototype global model with three countries, one sector and two factors of production is used to illustrate numerically some important features of national currencies and their exchange rates in a global CGE model. Note that the model is a theoretical one, whose equilibrium is not calibrated from a pre-determined database, but solved from an initial set of given factor endowments and parameter values.

In this model, each country uses a given amount of labour and capital to produce and trade one good. Each country purchases all three goods in consumption. Goods are assumed to be differentiated by country of origin. Two versions of the model are presented below: Model 1 is a conventional one with no national currencies and Model 2 includes national currencies and exchange rates.

Model 1: No national currency

This is a conventional model with no domestic currencies. The model can be specified by the six endogenous variable and equation groups (table 1).

The first five equation groups define five groups of endogenous variables. The last equation specifies a market clearing condition (MCC) for factors, which is used to determine the value of the only endogenous variable that is not defined by any equation, the factor price $P^{fac}_{(f,r)}$. Therefore, with equal numbers of endogenous variables and equations, the model should have a unique solution for all endogenous variables.

According to Walras' Law, the system is overdetermined: one of the prices is redundant. An endogenous price variable must be made exogenous so that it can be used as a numeraire to measure other prices against. Let the basic price of capital in Country A, $P^{fac}_{("cap","A")}$, be chosen as the numeraire and, therefore, the corresponding equation that specifies the MCC for that factor can be removed. This completes the core equation system of the model.

Table 1 **Core variables and equations for prototype Model 1**

1. CES demand of country r for factor f

$$Q^{fac}_{(f,r)} = X_{(r)} \left(\frac{\lambda_{(f)} P^{fac}_{\_f(r)}}{P^{fac}_{(f,r)}} \right)^{\sigma} \qquad (f = cap, lab;\ r = A, B, C)$$

where $P^{fac}_{\_f(r)}$ is a CES price index for the composite factor of country r,

$$P^{fac}_{\_f(r)} = \left(\sum_f \lambda_{(f)}{}^{\sigma} P^{fac}_{(f,r)}{}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \qquad (r = A, B, C)$$

2. Basic price of the output from country r

$$P^{b}_{(r)} = \frac{1}{X_{(r)}} \sum_f P^{fac}_{(f,r)} Q^{fac}_{(f,r)} \qquad (r = A, B, C)$$

3. Output of country r

$$X_{(r)} = \sum_s Q_{(r,s)} \qquad (r = A, B, C)$$

4. *CES demand of country* r *for the output from country s*

$$Q_{(r,s)} = \frac{Y_{(s)}}{P^{b}_{(r)}} \frac{\delta_{(r,s)}{}^{\sigma} P^{b}_{(r)}{}^{1-\sigma}}{\sum_i \delta_{(i,s)}{}^{\sigma} P^{b}_{(i)}{}^{1-\sigma}} \qquad (r, s = A, B, C)$$

5. Income of country r

$$Y_{(r)} = \sum_f P^{fac}_{(f,r)} X^{fac}_{(f,r)} \qquad (r = A, B, C)$$

6. Market clearing condition for factor f in country r

$$X^{fac}_{(f,r)} = Q^{fac}_{(f,r)} \qquad (f = cap, lab;\ r = A, B, C)$$

where $X^{fac}_{(f,r)}$ is country r's endowment of factor f.

Assume the following parameter values: $\lambda_{(f)} = 1/2$, $\delta_{(r)} = 1/3$ and $\sigma = 0.99$. For given endowments, $X^{fac}_{(f,r)}$ (listed in the factor supply part of the table), the general equilibrium solution is in table 2.[5]

Table 2 **General equilibrium results: Model 1 with no national currencies**

		A	B	C
Factor supplies	$X^{fac}_{("cap",s)}$	0.8	1.6	1.5
	$X^{fac}_{("lab",s)}$	2.0	1.8	1.2
Factor prices	$P^{fac}_{("cap",s)}$	1	0.50	0.53
	$P^{fac}_{("lab",s)}$	1.19	1.32	1.99
Income	$Y_{(s)}$	3.18	3.18	3.18
Output	$X_{(s)}$	1.59	1.75	1.27
Basic prices	$P^{b}_{(s)}$	2.00	1.82	2.51
Domestic prices	$P^{b}_{("A")}$	2.00	2.00	2.00
	$P^{b}_{("B")}$	1.82	1.82	1.82
	$P^{b}_{("C")}$	2.51	2.51	2.51
Consumption	$Q_{("A",s)}$	0.53	0.53	0.53
	$Q_{("B",s)}$	0.58	0.58	0.58
	$Q_{("C",s)}$	0.42	0.42	0.42

Source: The author's simulation.

This solution produces a model database. The output supply and demand data are shown in table 3. The diagonal of this table is the output consumed by the corresponding producing countries. The off-diagonal is a trade matrix: the rows and columns show the exports and imports of each country. Note that without NER, the prices of exports and imports are assumed to be denominated in a common currency, which is implicit and need not be specified. In this table, the row totals and column totals are all equal, implying a balanced external account for each country.

5 Other indicators, such as exports and imports, can be readily derived from the core variables and are not presented in the table.

Table 3 **Model 1 database: Output supplies and demands**

	A	B	C	Supply
A	1.06	1.06	1.06	3.18
B	1.06	1.06	1.06	3.18
C	1.06	1.06	1.06	3.18
Demand	3.18	3.18	3.18	

Source: The author's simulation.

Model 2: With national currencies and bilateral exchange rates

Now introduce a variable for the value of domestic currency in each country, $C_{(r)}$. Bilateral foreign exchange rates can then be defined as a ratio of the currencies of two trading countries as follows,

$$(6) \qquad r^{ex}_{(r,s)} = \frac{C_{(s)}}{C_{(r)}} \qquad (r,\, s = A,\, B,\, C)$$

This NER is defined as the domestic currency price of a unit of foreign currency. Note also that $r^{ex}_{(r,s)}$ equals unity when $r = s$. In a model with three currencies, the value of one currency is redundant because it can be determined by the values of the other two currencies. This means that if one currency is normalised to unity as a benchmark, the values of the other two currencies can be measured as the units of the benchmark currency. In this model, the currency of country A is normalised to unity, $C_{("A")} \equiv 1$. Note also that bilateral exchange rates are independent of the value of $C_{("A")}$.

When introducing domestic currencies and NERs, the basic prices of goods are denominated in the currencies of their countries of origin. In this model, the basic prices for imports in the demand function (equation 4 in table 1) need to be converted into the domestic prices of the importing countries, $P^{d}_{(r)}$, using bilateral exchange rates,

$$(7) \qquad Q_{(r,s)} = \frac{Y_{(s)}}{P^{d}_{(r)}} \frac{\delta_{(r,s)}{}^{\sigma} P^{d}_{(r,s)}{}^{1-\sigma}}{\sum_{i} \delta_{(i,s)}{}^{\sigma} P^{d}_{(i,s)}{}^{1-\sigma}} \qquad (r,\, s = A,\, B,\, C)$$

Note that $P^{d}_{(r,s)}$ is the domestic price of country s for the output of country r. This price is converted from country r's currency to country s's currency, using $r^{ex}_{(r,s)}$,

$$(8) \qquad P^{d}_{(r,s)} = r^{ex}_{(r,s)}\, P^{b}_{(r)} \qquad (r,\, s = A,\, B,\, C)$$

Introducing a new variable, $r^{ex}_{(r,s)}$, and a new equation to define this variable does not change the balance between the endogenous variables and model equations in the model closure.

Assume initially that, all $C_{(r)} = 1$; then $r^{ex}_{(r,s)}$ must all be equal to unity too. As a result, all results remain unchanged.

However, the introduction of national currencies and NERs allows the database of each country to be expressed in its own currency. Goods traded between countries now need to be converted from foreign currency prices into domestic currency prices, using bilateral exchange rates.

This leads to an important change in the model's numeraire. The newly introduced currencies can now be used as a national numeraire for each country. In the case of the above model, the two currencies, $C_{("B")}$ and $C_{("C")}$, become the numeraires for Countries *B* and *C*. As $C_{("A")}$ has already been chosen as a benchmark currency, the old global numeraire, $P^{fac}_{("cap","A")}$, the rental price of capital in Country *A*, remains the global numeraire for this closure. However, if $C_{("B")}$ and $C_{("C")}$ are swapped with $P^{d}_{("B","B")}$ and $P^{d}_{("C","C")}$, $P^{d}_{("B","B")}$ and $P^{d}_{("C","C")}$ become national numeraires for Countries *B* and *C*. Moreover, the old global numeraire $P^{fac}_{("cap","A")}$ is turned into a national numeraire for Country *A* only.[6] In this model there are multiple numeraires, each of which determines the price level for each country. This is a fundamental shift in the price structure of such models. It makes a global model truly international, because the domestic price system of each country is independent from each other and foreign trade and investment between countries have to be mediated by bilateral NERs.

The use of national currencies as national numeraires has another advantage for database manipulation: it becomes easier to transform a model's database from the old one, denominated in a single currency, into a new one, denominated in multiple national currencies. Such a transformation can be achieved by simply resetting the units of individual countries' currencies. For example, if the value of Country *B*'s currency is one half of Country *A*'s currency, setting $C_{("B")} = 2$ would produce a database, in which all Country *B*'s prices are double their original levels with no change in other variables. Each country's supplies of its output to different countries and the demand for outputs from different countries are now valued at the different prices, which should be presented in separate tables (table 4).

In the supply table in table 4, each row represents the supply of a country for domestic use or export, valued at its own price. The row sum is the supply of a country's output. In the demand table, each column represents the demand of a country for domestic use or import, valued at its own price. The values in the demand table are converted from the values of the supply table using the NERs. It can also be seen in the table the change in the relative values between currencies does not alter the underlying balance between supply and demand: all countries external accounts are still balanced. This example shows that, if the ratios between currency units are known, a new database can be easily complied by imposing such ratios in the model.

6 Alternatively, $P^{fac}_{("cap","A")}$ can be swapped with $P^{d}_{("A","A")}$, so that the latter is also a national numeraire.

Table 4 **Model 2 database: Output supply and demand tables**

Supply table (dom price)

	A	B	C	Supply
A	1.06	1.06	1.06	3.18
B	2.12	2.11	2.12	6.35
C	1.06	1.06	1.06	3.19

Demand table (dom price)

	A	B	C
A	1.06	2.12	1.06
B	1.06	2.11	1.06
C	1.06	2.12	1.06
Demand	3.18	6.35	3.19

Bilateral exchange rate

	A	B	C
A	1	2.00	1.00
B	0.50	1	0.50
C	1.00	2.00	1
Currency	1	2	1

Source: . The author's simulation.

A national numeraire provides a powerful tool to model the effects of some institutional arrangements on national price levels. For example, if the CPI is targeted, or any other domestic price is regulated or constrained, this price variable can be set as exogenous as a national numeraire. Such a closure option cannot be implemented in a global CGE model without currencies and NERs, because fixing a nominal price would violate the fundamental requirement of homogeneity for a CGE model.

A numerical example for comparing results from the two models

In the following, the role of currencies and NERs in the world CGE model is tested with a sample simulation. The test is a 20 per cent tariff imposed by Country *B* on its imports from Countries *A* and *B*. Two closures are used in this simulation to compare results. In the first closure, all currencies are set as exogenous so that the model behaves just like a conventional model with no NERs. In the second closure, it is assumed that the domestic basic prices of all three countries, $P^{b}_{(r)}$, are fixed at their pre-tariff levels (table 2) and their currencies except $C_{("A")}$, are set as endogenous so that their respective NERs can adjust. The results from the two simulations are compared in table 5 to see the impacts that currencies and NERs have on a world CGE model. The real exchange rate, $r^{rex}_{(r,s)}$, in the table is defined as,

$$(9) \qquad r^{rex}_{(r,s)} = \frac{r^{ex}_{(r,s)}\, P^{b}_{(r)}\,(1+t_{(r,s)})}{P^{b}_{(s)}} = \frac{P^{d}_{(r,s)}\,(1+t_{(r,s)})}{P^{b}_{(s)}} \qquad (r, s = A, B, C)$$

Table 5 **Results comparison: a 20% tariff in Country B with fixed and endogenous currencies**

		Fixed currencies			Endogenous currencies		
		A	B	C	A	B	C
Factor supply	$X^{fac}_{("cap",s)}$	0.8	1.6	1.5	0.8	1.6	1.5
	$X^{fac}_{("lab",s)}$	2.0	1.8	1.2	2.0	1.8	1.2
Factor price	$P^{fac}_{("cap",s)}$	1	0.53	0.53	1.00	0.50	0.53
	$P^{fac}_{("lab",s)}$	1.19	1.41	1.99	1.19	1.32	1.99
Income	$Y_{(s)}$	3.18	3.81	3.18	3.18	3.57	3.19
Output	$X_{(s)}$	1.59	1.75	1.27	1.59	1.75	1.27
Basic price	$P^{b}_{(s)}$	2.00	1.94	2.51	2	1.81	2.51
Currency	$C_{(s)}$	1	1	1	1	0.94	1.00
Nominal Exchange rate	$r^{ex}_{("A",s)}$	1	1.00	1.00	1	0.94	1.00
	$r^{ex}_{("B",s)}$	1.00	1	1.00	1.07	1	1.07
	$r^{ex}_{("C",s)}$	1.00	1.00	1	1.00	0.94	1
Tariff rate	$t_{("A",s)}$	0%	20%	0%	0%	20%	0%
	$t_{("B",s)}$	0%	0%	0%	0%	0%	0%
	$t_{("C",s)}$	0%	20%	0%	0%	20%	0%
Domestic price	$P^{d}_{("A",s)}(1+t_{("A",s)})$	2.00	2.40	2.00	2.00	2.25	2.00
	$P^{d}_{("B",s)}(1+t_{("B",s)})$	1.94	1.94	1.94	1.93	1.81	1.93
	$P^{d}_{("C",s)}(1+t_{("C",s)})$	2.51	3.01	2.51	2.51	2.83	2.51
Consumption	$Q_{("A",s)}$	0.53	0.53	0.53	0.53	0.53	0.53
	$Q_{("B",s)}$	0.55	0.65	0.55	0.55	0.65	0.55
	$Q_{("C",s)}$	0.42	0.42	0.42	0.42	0.42	0.42
Real exchange rate	$r^{rex}_{("A",s)}$	1	1.24	0.80	1	1.24	0.80
	$r^{rex}_{("B",s)}$	0.97	1	0.77	0.97	1	0.77
	$r^{rex}_{("C",s)}$	1.25	1.55	1	1.25	1.55	1

Source: The author's simulation.

In the first case with all prices adjusting, Country *B*'s domestic prices of imports are 20 per cent higher than their world prices, as might be expected. Factor and output prices increase less than 20 per cent, because of the imperfect substitution between products. With national currencies and NERs fixed, all adjustments in IRPs or RERs are carried out by national prices.

In the second case, all domestic basic prices are fixed while all currencies, except $C_{("A")}$, and their bilateral NERs are adjustable. For example, in Country *B*, the world price of export from Country *A* is 2.00. As Country *B*'s basic price is fixed, the domestic price of this import cannot increase by a full 20 per cent to 2.40. As a result, Country *B*'s currency has to appreciate so that its domestic prices of imports are 20 per cent higher than their world prices if converted into their foreign currencies. The results show that only country *B*'s domestic prices, including export prices, are affected. The changes in other countries' price and currency variables remain the same as that in the first case. More importantly, the results on the quantitative variables are identical in two simulations. This is because, with flexible national currencies, all countries are still able to adjust their IRPs. As a result, the same adjustments in bilateral RERs can still be made. If RERs remain unchanged, the demand and supply decisions by producers and consumers are not expected to change. This explains the same quantitative results for the two closures.

The above results confirm that introducing national currencies does not alter the homogeneity property of the new model. The key for understanding these results lies in the role that RERs play in this model, which captures IRPs between countries. When a policy change is imposed in a country, the costs of its production may be affected, leading to a change in its national price level, relative to other countries. To restore external balance, adjustments need to be made in the country's IRPs, indicated by changes in its RERs with other countries. For a given policy change, the adjustments in a country's RERs should always be the same. This requires a country's RER to be fully flexible so that such required adjustments can always be made. Otherwise, the country's external account could not be balanced, which is a necessary condition for an equilibrium solution.

The example shows that the same adjustments in RERs can be made through different price adjustment mechanisms. Introducing currencies and NERs into a world model make alternative price adjustments in a world CGE model possible. It also allows some domestic price or price index to be fixed to reflect the features of certain institutional arrangements and facilitates the interpretation of results.

A global CGE model with bilateral exchange rates

In this section, a global CGE model is used to demonstrate what is required to transform a conventional CGE model into a model with national currencies and bilateral NERs. The theoretical structure of this model is similar to that of the GTAP model, because it draws its data from a GTAP database (version 7) (Badri and Walmsley 2008), but it consists of a much simpler core equation system. An important advantage of using a simple structured model is its clarity and transparency: it is easy to locate the equations that are required for modification. A full list of this model's core equations can be found in Zhang (2013).

As mentioned earlier, introducing national currencies and bilateral NERs requires a fully bilateral database. That is, any nominal flow in the database must be convertible from one country's price to another's through a bilateral exchange rate. Not all databases meet this requirement. For example, the popular GTAP database is not fully bilateral due to two distinctive features.

1. Transport margins are described in two matrixes: a 4-dimensional demand (import) matrix and an aggregated 2-dimensional supply (export) matrix. It is therefore not possible to identify the source of a margin service that is used to supply a good to an importing country.
2. The trade deficit in a country is assumed to be financed by a *net* investment of foreign savings, which are implicitly assumed to be collected by a "world bank". With this net measure, the foreign savings, denominated in their respective foreign currencies, cannot be converted to investment flows, denominated in the receiving country's currency.

To introduce national currencies and bilateral exchange rates, additional data work is required. The objective is (a) to combine and extend the two margin matrixes to form a complete 5-dimensional transport margin matrix, and (b) to create a 2-dimensional gross saving and investment matrix from two vectors of the aggregate national savings and the aggregate national investments, implied in the current database. The database extension is exposed in appendix A. Once the database is ready, the new variables and equations can be introduced and the required modification of the model equations can be carried out. The equation modification is detailed in appendix B.

Test simulations with the new model

In the following, a policy simulation is conducted to test for homogeneity and the function of the new price adjustment mechanisms. The policy is a hypothetical rise of the US tariffs to 45 per cent on all imports from China. This simulation is repeated under the following five scenarios, each of which is characterised by a different closure. The scenarios are:

1. All national currencies are set as exogenous and a world price index for factors is used as a numeraire. This closure is equivalent to that of a conventional global CGE model without exchange rates.
2. All national currencies are set as endogenous, except the US dollar, which is used as the benchmark currency, and all national GDP deflators are set as exogenous.
3. Extending on Scenario 2, the Chinese Yuan is pegged to the US dollar, and China's GDP deflator is allowed to adjust.
4. Extending on Scenario 2, China's wage rate for unskilled labour is fixed and its GDP deflator is set as endogenous.
5. All national CPIs are fixed, while all national currencies, except the US dollar, are set as endogenous.

Across all scenarios, results for real GDP (column 1 in table 6) are the same, as expected, because the theory introduced should not affect allocation decisions since the changes in bilateral RERs are the same for all scenarios. Both countries suffer from the losses of bilateral trade: China and US real GDP fall by 0.72 and 0.34 per cent, respectively (column 1). This verifies the homogeneity of the model. The following analysis concentrates on interpreting the different price adjustment mechanisms that are inherent to each closure.

In Scenario 1, as national currencies are fixed so that all bilateral exchange rates are held constant, the national price levels have to adjust to the policy changes. For example, China's GDP deflator (pGDP) falls 4.73 per cent while the US domestic price level increases 0.88 per cent (column 2). When all domestic prices are flexible, bilateral RERs for all countries can adjust to balance their external account. The required changes in China's bilateral RERs are shown in column 3. The results for the next four scenarios show how GDP deflators and national currencies interact with each other to ensure that required adjustments in RERs can be made so that allocation decisions remain unchanged.

Table 6 **Results for real GDP, GDP deflators and exchange rates**

	Scenario 1 (rEX = 0)			Scenario 2 (pGDP = 0)		Scenario 3 (rEX(chn,usa) = 0)		Scenario 4 (pL(chn) = 0)		Scenario 5 (CPI = 0)	
	qGDP (r) (1)	pGDP (r) (2)	rREX (r,chn) (3)	pGDP (r) (4)	rEX (r,chn) (5)	pGDP (r) (6)	rEX (r,chn) (7)	pGDP (r) (8)	rEX (r,chn) (9)	pGDP (r) (10)	rEX (r,chn) (11)
1 aus	0.03	0.03	4.99	0	4.99	0	-0.85	0	5.58	0.04	4.00
2 nzl	0.02	0.13	5.11	0	5.11	0	-0.74	0	5.70	0.05	4.10
3 chn	-0.72	-4.73	0	0	0	-5.57	0	0.56	0	-0.90	0
4 jpn	0.00	0.13	5.10	0	5.10	0	-0.74	0	5.69	0.08	4.07
5 kor	0.03	0.10	5.07	0	5.07	0	-0.78	0	5.66	0.14	3.97
6 twn	-0.01	0.20	5.18	0	5.18	0	-0.67	0	5.77	0.08	4.15
7 idn	0.03	0.47	5.46	0	5.46	0	-0.40	0	6.05	0.09	4.42
8 mys	0.05	1.15	6.18	0	6.18	0	0.27	0	6.77	0.59	4.60
9 tha	0.04	0.67	5.67	0	5.67	0	-0.21	0	6.26	0.26	4.44
10 ind	0.06	0.54	5.53	0	5.53	0	-0.34	0	6.12	0.07	4.50
11 can	0.06	1.45	6.49	0	6.49	0	0.57	0	7.09	0.27	5.24
12 usa	-0.34	0.88	5.89	0	5.89	0	0	0	6.49	0.12	4.81
13 bra	0.02	0.36	5.35	0	5.35	0	-0.52	0	5.93	0.03	4.36
14 eun	0.01	0.12	5.10	0	5.10	0	-0.75	0	5.69	0.04	4.11
15 rus	0.05	0.12	5.09	0	5.09	0	-0.76	0	5.68	0.12	4.01
16 zaf	0.03	0.20	5.17	0	5.17	0	-0.68	0	5.76	0.08	4.14
17 row	0.05	0.62	5.62	0	5.62	0	-0.26	0	6.21	0.17	4.48

Note: Nominal and real exchange rates are bilateral. Shown in the table are only the results for China's bilateral foreign exchange rates.

Source: The author's simulation.

In Scenario 2, national price levels are assumed to be fixed while national currencies are made flexible. In this case, bilateral exchange rates become fully adjustable so that the same results on RER and real GDP can still be achieved. The relations between the changes in RER and NER and price levels can be seen in the following definition for RER in its relative form (percentage changes),

$$(9) \qquad r^{rex}_{(r,s)} = r^{ex}_{(r,s)} + p^{gdp}_{(r)} - p^{gdp}_{(s)} \qquad (r, s \in \text{REG})$$

It is clear that when GDP deflators are fixed, the adjustment in RER is carried out fully by flexible NER. As a result, the changes in NERs should be exactly equal to the changes in RERs, $r^{rex}_{(r,s)} = r^{ex}_{(r,s)}$. This is confirmed by the results for China's NER in column 5, which are exactly the same as its results on RERs shown in column 3.

In Scenario 3 when the Chinese Yuan is pegged with the US dollar, as Chinese domestic price level is made adjustable together with all other currencies and their bilateral exchange rates with the Yuan, the same results on RERs and real GDP can still be achieved. Note that, as the US dollar is fixed, if the bilateral China-US exchange rate is fixed, the value of Chinese Yuan must also be held constant. However, China's bilateral exchange rates with other countries are still adjusted through changes in the values of other currencies. In this case, China's bilateral exchange rates with other currencies (column 7 in table 6) need to fall more to reach the adjustments in its RERs required for equilibrium. As a result, China's domestic price level (GDP deflator) also need to fall more (5.57 per cent, column 6) than what is required (4.47 per cent, column 1) when all bilateral exchange rates are held constant by their fixed national currencies. In this case, the required adjustments in RERs can still be made by China's flexible exchange rates with other currencies and China's own GDP deflator, $r^{rex}_{(r,s)} = r^{ex}_{(r,s)} - p^{gdp}_{(s)}$.

In Scenario 4, when China's wage rate for unskilled labour is fixed, its GDP deflator needs to be set as endogenous because this is a swap between national numeraire variables. As shown in the table, China's GDP deflator is 0.56 per cent higher than otherwise, which is the result of a fixed wage. This is the opposite direction required for China's national price level to reach equilibrium. To correct this price increase, Chinese Yuan has to be depreciated more (0.56 per cent) to induce the required adjustments in its RERs. Again, this results can be measured in the same way as in Scenario 3, $r^{rex}_{(r,s)} = r^{ex}_{(r,s)} - p^{gdp}_{(s)}$.

In Scenario 5, when a country's CPI is fixed, its GDP deflator is tied down by its fixed CPI and cannot be fully adjusted to the required level. A flexible NER is therefore needed to complete the adjustment required in the RER. In this case, changes in both national prices and national currencies contribute to the changes in RERs, $r^{rex}_{(r,s)} = r^{ex}_{(r,s)} + p^{gdp}_{(r)} - p^{gdp}_{(s)}$.[7]

The above simulation results confirm that the inclusion of national currencies and bilateral exchange rates does not alter the nature of a world CGE model. Its homogeneity property and its nature as a real economy model remain intact. However, with currencies and NERs as variables, this model offers alternative closure options to allow for national currencies and NERs to play a central role in relative price adjustments. It also becomes possible for individual national economies to be modelled in terms of their own national currencies, rather than in an implicit "international currency". Although they are not expected to have real impacts on the model results, these closure options enhance our understanding of the alternative price adjustment mechanisms, in particular, what role currencies and exchange rates may play in driving a country's price system to a new equilibrium.

7 Note that the post-simulation calculations using these equations may not be accurate, due to linearization errors.

Concluding remarks

This paper proposes an approach to introduce national currencies and NERs in global CGE models, using theory to link the value of national currency to national price level in each country. Bilateral NERs are introduced as the relative prices of national currencies. The NER is used to adjust a country IRPs, reflected in bilateral RERs, to balance its external account. An advantage of using bilateral NERs over effective rates is its clarity and transparency. It removes any ambiguity in inter-country relations in a world model and allows these relations to be defined on a bilateral basis. This also makes it possible to conduct policy analyses that might target bilateral relations between two countries.

A prototype model is used to illustrate the role of national currencies and bilateral NERs in a multi-country model. Although it does not affect real impacts in the model, the introduction of national currencies and NERs changes the units of measurement that underlie the value flows of the model database. Trade between countries has to be conducted through currency exchange using bilateral NERs. These changes transform the traditional global CGE model into a model made of independent countries that control their own prices. In such a model, the NERs play a central role in adjusting relative prices between countries to balance their external accounts.

A simple-structured global model is also used to show what is required to introduce national currencies and NERs in a conventional global model, such as GTAP. A key requirement for introducing bilateral NERs is to bilateralise the model database fully so that the NERs can be used to convert the prices of goods and services, denominated in one country's currency to another. Some simulations are conducted to test the properties of the new models under alternative price adjustment mechanism, involving endogenous national currencies and NERs. The test results show the new model preserves the homogeneity properties of a CGE model. They also show that, with currencies and NERs, the new model can accommodate alternative price adjustment mechanisms, which could not be captured in a conventional model. This possibility means that the effects of policies that restrict price movements can be incorporated as part of the modelling environment.

In this paper, national currencies are not introduced as a financial asset. However, the inclusion of national currencies and NERs in a world model can be seen as a first step in that direction. For example, the creation of independent national economies in a world model makes it possible to model the potential effects of monetary policies on relevant variables on a national basis, a prerequisite to a new type of CGE model, which incorporates financial assets. In fact, the currency as introduced in this paper already has its own value, and is, therefore, ready to be used as a financial asset in a financial CGE model. In such a model, NERs and uncovered interest parity can be used to guide the allocation of investment and savings across countries. Such a model will be valuable for analysing a wide range of issues that are currently beyond the reach of conventional real-economy models, including the type of events that underlie the 2008 global financial crisis.

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Appendix A: Database extension

In appendix A, the constructions of a complete transport margin matrix and a saving-investment matrix are discussed.

Construct a complete transport margin matrix

A new bilateral transport margin matrix can be created by combining the two existing transport margin matrixes. To create this matrix, a destination dimension is added to the existing 2-dimensional matrix of margin exports, $V^{mexp}_{(m,t)}$, to create a 3-dimensional matrix $V^{mrg}_{\_cs(t,m,r)}$. It shows the value of margin transport m exported from source country t to the destination country r. Without new data on the destinations of margin transports, it is assumed that the export of a country's transport services is equally shared among all the margin user countries, exclusive of itself.

$$V^{mrg}_{\_cs(t,m,r)} = s^{mexp}_{(m,t,r)} \sum_{c} \sum_{s} V^{mimp}_{(m,c,r,s)} \qquad (m = \text{MCOM};\ t,\ r = \text{REG})$$

where $V^{mimp}_{(m,c,r,s)}$ is the existing margin import matrix and $s^{mexp}_{(m,t,r)}$ is the share of transport margin m exported from country t to country r, derived from the margin export matrix, $V^{mexp}_{(m,t)}$.

$$s^{mexp}_{(m,t,r)} = \frac{V^{mexp}_{(m,t)}}{\sum_{s} V^{mexp}_{(m,s)}} \qquad (m = \text{MCOM};\ t,\ r = \text{REG})$$

Note that $t \neq r$ for individual countries to avoid two-way trade in margins, except for those multi-country regions. As $s^{mexp}_{(m,t,r)}$ is derived from the 2-dimensional margin export matrix, the resulting matrix $V^{mrg}_{\_cs(t,m,r)}$ is not consistent with the aggregates in all three dimensions. A RAS method is therefore required to adjust the matrix so that it can be aggregated back to the two original matrixes: the 2-dimensional margin supply matrix and the 4-dimensional margin demand matrix.

$$\sum_{r} V^{mrg}_{\_cs(t,m,r)} = V^{mexp}_{(m,t)} \qquad (m = \text{MCOM};\ t,\ r = \text{REG})$$

$$\sum_{t} V^{mrg}_{\_cs(t,m,r)} = \sum_{c} \sum_{s} V^{mimp}_{(m,c,r,s)} \qquad (m = \text{MCOM};\ t,\ r = \text{REG})$$

Once confirmed, the matrix of aggregate margin exports, $V^{mrg}_{\_cs(t,m,r)}$, can then be allocated to each imported good in a new 5-dimensional matrix $V^{mrg}_{(t,m,c,r,s)}$, to replace the 4-dimensional matrix of margin imports, $V^{mimp}_{(m,c,r,s)}$,

$$V^{mrg}_{(t,m,c,r,s)} = s^{mimp}_{(m,c,r,s)}\ V^{mrg}_{\_cs(t,m,r)} \qquad (m = \text{MCOM};\ c = \text{COM};\ t,\ r,\ s = \text{REG})$$

where $s^{mimp}_{(m,c,r,s)}$ is the shares of margin imports,

$$s^{mimp}_{(m,c,r,s)} = \frac{V^{mimp}_{(m,c,r,s)}}{\sum_i \sum_g V^{mimp}_{(m,i,r,g)}} \qquad (m = \text{MCOM};\ c = \text{COM};\ r,\ s = \text{REG})$$

It can be verified that the original *f*.o.*b*. export matrix plus the newly derived transport margins should be equal to the original *c.i.f.* import matrix,

$$V^{cif}_{(c,r,s)} = V^{fob}_{(c,r,s)} + \sum_t \sum_m V^{mrg}_{(t,m,c,r,s)} \qquad (c = \text{COM};\ r,\ s = \text{REG})$$

These new transport margin and trade matrixes have clear bilateral price structures, which allows for inter-country price conversion using bilateral NERs.

Create a saving-investment matrix

The other data work is to construct a bilateral saving-investment matrix so that the value of savings can be converted from one country's currency to another country's currency using bilateral NERs.

In the GTAP-style global model, a country's aggregate investment and savings are determined by the database. The trade deficit of a country ($M - X$) is assumed to be financed by the inflow of foreign investment, which is equal to the aggregate investment net of domestic savings ($I - S$),

$$M - X = I - S$$

In a global model, the above equality can be expressed in the matrix form as follows,

$$\sum_r (V^{trd}_{(r,s)} - V^{trd}_{(s,r)}) = \sum_r (Y^{inv}_{(r,s)} - Y^{inv}_{(s,r)}) \qquad (s = \text{REG})$$

In this expression, $V^{trd}_{(r,s)}$ is a matrix that traces the trade flows from source r to destination s. This matrix can be derived from the original trade matrix and the transport margin matrix developed above.

$$V^{trd}_{(r,s)} = \sum_c V^{fob}_{(c,r,s)} + \sum_m \sum_c \sum_g V^{mrg}_{(r,m,c,g,s)} \qquad (r,\ s = \text{REG})$$

The other matrix, $Y_{(s,r)}^{inv}$, is a saving-investment matrix. Although it is not readily available from the existing database, we know that the row sum of this matrix should be equal to the aggregate investments used by countries, while the column sum should be equal to the aggregate savings contributed by countries. These national aggregates can be derived from the original database.

In the absence of foreign investment data, bilateral net imports could be used as a proxy for the saving-investment matrix because the net investment inflows must be equal to the net import flows in equilibrium.

$$Y_{(r,s)}^{inv} = If\,(V_{(r,s)}^{trd} - V_{(s,r)}^{trd} > 0,\ 0) \qquad (r,\ s = \text{REG})$$

Note that $Y_{(r,s)}^{inv}$ takes only the positive values from the net import calculation. As a result, the above *MX-IS* equality holds not only at the national, or aggregate, level, but also at the bilateral level,

$$V_{(r,s)}^{trd} - V_{(s,r)}^{trd} = Y_{(r,s)}^{inv} - Y_{(s,r)}^{inv} \qquad (r,\ s = \text{REG})$$

This saving-investment matrix satisfies all the balancing requirements and, therefore, is suitable for being used in a model incorporated with bilateral NERs.

Appendix B: Equation system modification

In Appendix B, the new variables and equations are introduced and the required modification of the model equations are outlined. The equation system modification is based on a conventional global CGE model without NREs, presented in Zhang (2013). The core equation system of the new global model with national currencies and NERs can be found in table B.1 and the sets used to define variables and equations are listed in table B.2.

First, a new variable for bilateral NERs $r^{ex}_{(r,s)}$ is introduced in the model structure and defined in equation 27 of table B.1 [8] as a ratio of two national currencies,

$$r^{ex}_{(r,s)} = \frac{C_{(s)}}{C_{(r)}} \qquad (r,s \in \text{REG})$$

where $C_{(r)}$ and $C_{(s)}$ are the currencies of foreign country *r* and home country *s*, respectively. To keep the original model closure intact, all national currencies are introduced initially as an exogenous variable. This assumption can be relaxed in alternative closures.

When national currencies are introduced, the goods and services produced in each country are valued in terms of its own currency. When these goods and services are exported, their prices are treated by importing countries as the world prices. Therefore, NERs are used by importing countries to convert the world prices of their imports into their domestic prices. Similarly, if a country receives foreign investment, the foreign currency value of such investment must be converted into a domestic currency value using a NER before it can be used in the domestic economy.

The modification of the model's core equation system is related mainly to two areas: the conversion of the prices of traded goods and services (equation 11 in section 2) and the conversion of the values of savings and investment (equations 22-26 in section 5).

Modify transport margin and trade price equations

First, the prices of transport margins need to be converted from the currencies of margin-exporting countries to the currencies of goods-exporting countries, and then to the currencies of the goods-importing countries using their respective bilateral NERs (equation 11 in section 2).

8 The following equation numbers all refer to those in table B.1.

The CES price of composite margin transport service is denominated in country *r*' currency. It is converted by an exchange rate from the price of margin exporting country *t* to the price of margin importing country *r*,

$$P^{mimp}_{(m,i,r,s)} = CES\,(P_{(m,t_1,\text{"}dom\text{"})}\, r^{ex}_{(t_1,r)},\ \ldots,\ P_{(m,t_n,\text{"}dom\text{"})}\, r^{ex}_{(t_n,r)}) \qquad (m\in\text{MCOM};\ i\in\text{COM};\ r,s\in\text{REG})$$

The CES demand of importing country *r* for margin good *m*, used in the export of good *i* from country *r* to country *s*,

$$Q^{mrg}_{(t,m,i,r,s)} = CES\,(P_{(m,t,\text{"}dom\text{"})}\, r^{ex}_{(t_1,r)},\ P^{mimp}_{(m,i,r,s)},\ Q^{mimp}_{(m,i,r,s)}) \qquad (m\in\text{MCOM};\ i\in\text{COM};\ t,r,s\in\text{REG})$$

where $Q^{mimp}_{(m,i,r,s)}$ is determined by a Leontief demand function of bilateral imports,

$$Q^{mimp}_{(m,i,r,s)} = Leontief\,(Q^{trd}_{(i,r,s)}) \qquad (m\in\text{MCOM};\ i\in\text{COM};\ r,s\in\text{REG})$$

Once the transport margin prices are defined, the c.i.f. price of import *i* can be defined as equal to its f.o.b. price, plus a margin mark-up (equation 3)

$$P^{cif}_{(i,r,s)} = P^{fob}_{(i,r,s)} + \frac{1}{Q^{trd}_{(i,r,s)}} \sum_m Q^{mimp}_{(m,i,r,s)}\, P^{mimp}_{(m,i,r,s)} \qquad (i\in\text{COM};\ r,s\in\text{REG})$$

Note that this price is denominated in exporting country *r*'s currency. When the goods are imported into country *s*, their foreign prices are converted into their domestic prices using respective bilateral NERs (equation 2),

$$P^{imp}_{(i,r,s)} = P^{cif}_{(i,r,s)}\,(1 + t^{imp}_{(i,r,s)})\, r^{ex}_{(r,s)} \qquad (i\in\text{COM};\ r,s\in\text{REG})$$

Modify saving-investment equations

The introduction of bilateral saving-investment matrix requires some modifications of the model equations. Ideally, a bilateral saving-investment matrix should be associated by a bilateral capital stock matrix so that the rate of return to capital can be defined on a bilateral basis. Without such a capital stock matrix, a CET function is chosen as a compromise for modelling the supply of savings for investment across countries.

The CET allocation of savings from country *r* to be invested in country *s* is defined in equation 24,

$$Y^{inv}_{(r,s)} = CET\,(Y_{(\text{"}inv\text{"},r)},\ R^{e}_{\_s(r)},\ R^{e}_{(r,s)}) \qquad (r,s\in\text{REG})$$

where $Y_{("inv",r)}$ is the total savings in country r, $R^{e}_{(r,s)}$ is the expected rate of return to investment from country r to country s and $R^{e}_{\_s(r)}$ is a CET price index for the expected rate of return for the aggregate savings from country r,

$$R^{e}_{\_s(r)} = CET\,(R^{e}_{(r,s_1)},\ \ldots,\ R^{e}_{(r,s_n)}) \qquad (r \in \mathrm{REG})$$

Bilateral real investments are defined as bilateral nominal savings divided by the price index of investment in the destination country s (equation 25),

$$Q^{inv}_{(r,s)} = \frac{Y^{inv}_{(r,s)}\, r^{ex}_{(r,s)}}{P^{t}_{\_is("inv",s)}} \qquad (r,s \in \mathrm{REG})$$

The sum of real investments from all sources should equal total investment used in a country (equation 26),

$$Q_{\_is("inv",r)} = \sum_{s} Q^{inv}_{(s,r)} \qquad (r \in \mathrm{REG})$$

This is a market equilibrium condition for real investment in country r, used to determine the general equilibrium value of the undefined variable $Y^{NFI}_{(r)}$, the inflow of foreign investment.

Exchange rates in parameter calibration

The introduction of exchange rates also changes the meaning or the units of nominal values of the model's database. In the original GTAP database, for example, all nominal value flows are denominated in US dollars. When NERs are introduced, the value flows of different countries have to be denominated by their own currencies, even if there is no change in the magnitudes of these flows. This is because the NERs can always be set as unity so that the existing database can still be used.

As the value flows are used in calibrating model's coefficients and parameters, the change in the value flows in the database fundamentally changes the way in which model parameter are calibrated. In the database, bilateral NERs need to be introduced as a new coefficient with unity as their initial values. When value flows are used in calibration, the relevant bilateral NERs must be used to convert the flows from one country's price to another country's price so that the value flows reflect the changes in the underlying national currencies.

Table B.1 The core equations of a global CGE model with bilateral exchange rates

1. Region and user's demands for goods (1-9)

(1) CES demand of region s for import i from region r

$$Q^{trd}_{(i,r,s)} = CES\,(P^{imp}_{(i,r,s)}, P_{(i,s,\text{"}imp\text{"})}, Q_{\_u(i,s,\text{"}imp\text{"})}) \qquad (i\in\text{COM};\ r,s\in\text{REG})$$

where $P_{(i,s,\text{"}imp\text{"})}$ is a CES price index for composite import i in region s.

(2) Domestic basic prices of import i from region r to region s, denominated in s's currency

$$P^{imp}_{(i,r,s)} = P^{cif}_{(i,r,s)}\,(1 + t^{imp}_{(i,r,s)}) \quad r^{ex}_{(r,s)} \qquad (i\in\text{COM};\ r,s\in\text{REG})$$

where $t^{imp}_{(i,r,s)}$ is the *ad valorem* rate of an import tariff.

(3) World *cif* price of import i from region r to region s, denominated in r's currency

$$P^{cif}_{(i,r,s)} = P^{fob}_{(i,r,s)} + \frac{1}{Q^{trd}_{(i,r,s)}} \sum_m Q^{mimp}_{(m,i,r,s)}\, P^{mimp}_{(m,i,r,s)} \qquad (i\in\text{COM};\ r,s\in\text{REG})$$

(4) World *fob* price of export i from region r to region s, denominated in r's currency

$$P^{fob}_{(i,r,s)} = P_{(i,r,\text{"}dom\text{"})}\,(1 + t^{exp}_{(i,r,s)}) \qquad (i\in\text{COM};\ r,s\in\text{REG})$$

where $t^{exp}_{(i,r,s)}$ is the *ad valorem* rate of an export tax.

(5) Regional total demand for good i from source s

$$Q_{\_u(i,r,s)} = \sum_u Q_{(i,u,r,s)} \qquad (i\in\text{COM};\ r\in\text{REG};\ s\in\text{SRC})$$

(6) CES demand for good i from source s by user u in region r

$$Q_{(i,u,r,s)} = CES\,(P^{t}_{(i,u,r,s)}, P^{t}_{\_s(i,u,r)}, Q_{\_s(i,u,r)}) \qquad (i\in\text{COM};\ u\in\text{USR};\ r\in\text{REG};\ s\in\text{SRC})$$

where $P^{t}_{\_s(i,u,r)}$ is a CES price index for composite good i for user u in region r,

$$P^{t}_{\_s(i,u,r)} = CES\,(P^{t}_{(i,u,r,\text{"}dom\text{"})}, P^{t}_{(i,u,r,\text{"}imp\text{"})}) \qquad (i\in\text{COM};\ u\in\text{USR};\ r\in\text{REG})$$

(7) Purchasers' price of good i from source s for user u in region r

$$P^{t}_{(i,u,r,s)} = P_{(i,r,s)}\,(1 + t^{dom}_{(i,u,r,s)}) \qquad (i\in\text{COM};\ u\in\text{USR};\ r\in\text{REG};\ s\in\text{SRC})$$

where $t^{dom}_{(i,u,r,s)}$ is the *ad valorem* rate of an indirect tax.

(8) Demand for composite good i by user u in region r

$$Q_{\_s(i,u,r)} = \begin{cases} Leontief\,(Q^{dom}_{(u,r)}) & (i\in\text{COM};\ u\in\text{IND};\ r\in\text{REG}) \\ f\,(E_{(u,r)}, P^{t}_{\_s(i_1,u,r)}, \ldots, P^{t}_{\_s(i_n,u,r)}) & (i\in\text{COM};\ u=hou;\ r\in\text{REG}) \\ f\,(E_{(u,r)}, P^{t}_{\_s(i,u,r)}) & (i\in\text{COM};\ u=gov;\ r\in\text{REG}) \\ Leontief\,(E_{(u,r)}, P^{t}_{\_is(u,r)}) & (i\in\text{COM};\ u=inv;\ r\in\text{REG}) \end{cases}$$

(9) Purchases' price index for composite goods for user u in region r

$$P^{t}_{\_is(u,r)} = \frac{1}{Q_{\_is(u,r)}} \sum_i Q_{\_s(i,u,r)}\, P^{t}_{\_s(i,u,r)} \qquad (u\in\text{USR};\ r\in\text{REG})$$

where $Q_{\_is(u,r)}$ is total demand for composite goods by user u in region r,

$$Q_{\_is(u,r)} = \sum_i Q_{\_s(i,u,r)} \qquad (u\in\text{USR};\ r\in\text{REG})$$

(continued)

Table B.1 (continued)

2. Industry's outputs, demands for and supplies of factors (10-15)

(10) Total demand for output from industry j in region r

$$Q^{dom}_{(j,r)} = \begin{cases} Q_{\_u(j,r,"dom")} + \sum_s Q^{trd}_{(j,r,s)} & (j \in \text{NCOM};\ r \in \text{REG}) \\ Q_{\_u(j,r,"dom")} + \sum_s Q^{trd}_{(j,r,s)} + \sum_i \sum_g \sum_s Q^{mrg}_{(r,j,i,g,s)} & (j \in \text{MCOM};\ r \in \text{REG}) \end{cases}$$

(11) CES demand of region r for margin good m from region t

$$Q^{mrg}_{(t,m,i,r,s)} = CES\,(P_{(m,t,"dom")}\ r^{ex}_{(t,r)},\ P^{mimp}_{(m,i,r,s)},\ Q^{mimp}_{(m,i,r,s)}) \qquad (m \in \text{MCOM};\ i \in \text{COM};\ t,r,s \in \text{REG})$$

where $P^{mimp}_{(m,i,r,s)}$ is a CES price index for composite margin good m, denominated in region r's currency,

$$P^{mimp}_{(m,i,r,s)} = CES\,(P_{(m,t_1,"dom")}\ r^{ex}_{(t_1,r)},\ \ldots,\ P_{(m,t_n,"dom")}\ r^{ex}_{(t_n,r)}) \qquad (m \in \text{MCOM};\ i \in \text{COM};\ r,s \in \text{REG})$$

and $Q^{mimp}_{(m,i,r,s)}$ is the demand for composite margin good m of non-margin good i, exported from region r to s, defined as a Leontief function of trade flows,

$$Q^{mimp}_{(m,i,r,s)} = Leontief\,(Q^{trd}_{(i,r,s)}) \qquad (m \in \text{MCOM};\ i \in \text{COM};\ r,s \in \text{REG})$$

(12) CES demand for factor f used by industry j in region r

$$Q^{fac}_{(f,j,r)} = CES\,(P^{tfac}_{(f,j,r)},\ P^{tfac}_{\_f\,(j,r)},\ Q^{dom}_{(j,r)}) \qquad (f \in \text{FAC};\ j \in \text{COM};\ r \in \text{REG})$$

where $P^{tfac}_{\_f\,(j,r)}$ is a CES price index for composite factor in industry j in region r,

$$P^{tfac}_{\_f\,(j,r)} = CES\,(P^{tfac}_{("land",j,r)},\ P^{tfac}_{("cap",j,r)},\ P^{tfac}_{("lab",j,r)}) \qquad (j \in \text{COM};\ r \in \text{REG})$$

(13) Purchasers' price for factor f in industry j of region r

$$P^{tfac}_{(f,j,r)} = P^{fac}_{(f,j,r)}\ (1 + t^{fac}_{(f,j,r)}) \qquad (f \in \text{FAC};\ j \in \text{COM};\ r \in \text{REG})$$

where $t^{fac}_{(f,j,r)}$ is the *ad valorem* rate of a tax on factor f used in industry j of region r.

(14) CET supply of land in industry j in region r (industry MEC for land)

$$X^{lnd}_{(j,r)} = CET\,(P^{*fac}_{("land",j,r)},\ P^{fac}_{\_j\,("land",r)},\ \bar{X}^{fac}_{("land",r)}) \qquad (j \in \text{COM};\ r \in \text{REG})$$

where $\bar{X}^{fac}_{("land",r)}$ is the exogenous supply of land in region r, $P^{*fac}_{("land",j,r)}$ is the *undefined* basic price of land and $P^{fac}_{\_j\,("land",r)}$ is a CET price index for composite land in region r,

$$P^{fac}_{\_j\,("land",r)} = CET\,(P^{*fac}_{("land",j_1,r)},\ \ldots,\ P^{*fac}_{("land",j_m,r)}) \qquad (r \in \text{REG})$$

(15) Basic price for good j from source s in region r

$$P_{(j,r,s)} = \begin{cases} \dfrac{1}{Q^{dom}_{(j,r)}} \left(\sum_i Q_{\_s\,(i,j,r)}\ P^{t}_{\_s\,(i,j,r)} + \sum_f Q^{fac}_{(f,j,r)}\ P^{tfac}_{(f,j,r)}\right) (1 + t^{prd}_{(j,r)}) & (j \in \text{COM};\ r \in \text{REG};\ s = dom) \\ CES\,(P^{imp}_{(j,k_1,r)},\ \ldots,\ P^{imp}_{(j,k_n,r)}) & (j \in \text{COM};\ r \in \text{REG};\ s = imp) \end{cases}$$

where $t^{prd}_{(j,r)}$ is the rate of a production tax.

3. Final user's income and expenditure (16-18)

(16) Household disposable income, government income and regional savings

$$Y_{(u,r)} = \begin{cases} \sum_{f \in \text{FAC}} P^{tinc}_{(f,r)}\ \bar{X}^{fac}_{(f,r)} & (u = hou;\ r \in \text{REG}) \\ (Total\ Tax\ Revenue) & (u = gov;\ r \in \text{REG}) \\ \sum_{i = hou,gov} Y_{(i,r)}\ s_{(i,r)} & (u = inv;\ r \in \text{REG}) \end{cases}$$

where $s_{(i,r)}$ is the saving rate for final user i (=hou, gov).

(continued)

Table B.1 (continued)

(17) Post-income tax price for factor *f* in region *r*

$$P^{tinc}_{(f,r)} = P^{*fac}_{\_j\,(f,r)}\,(1 - t^{inc}_{(f,r)}) \qquad (f \in \text{FAC};\ r \in \text{REG})$$

where $t^{inc}_{(f,r)}$ is the *ad valorem* rate of a tax on the income of factor *f*.

(18) Expenditure of final user *u* in region *r*

$$E_{(u,r)} = \begin{cases} Y_{(u,r)}\,(1 - s_{(u,r)}) & (u = hou,\ gov;\ r \in \text{REG}) \\ Y_{(u,r)} + Y^{NFI}_{(r)} & (u = inv;\ r \in \text{REG}) \end{cases}$$

4. Market equilibrium conditions for factors (19-21)

(19) Industry's MEC for land

$$X^{lnd}_{(j,r)} = Q^{fac}_{(\text{"}land\text{"},j,r)} \qquad (j \in \text{COM};\ r \in \text{REG})$$

(20) Regional MEC for labour and capital (=>$P^{fac}_{\_j\,(f,r)}$)

$$X^{fac}_{(f,r)} = \sum_j Q^{fac}_{(f,j,r)} \qquad (f = lab,\ cap;\ r \in \text{REG})$$

where $\bar{X}^{fac}_{(f,r)}$ is the exogenous supply of factor *f* (*lab*, *cap*) in region *r*.

(21) Basic prices for factor *f* (*lab*, *cap*) (PEC for labour and capital)

$$P^{fac}_{(f,j,r)} = P^{fac}_{\_j\,(f,r)} \qquad (f = lab,\ cap;\ j \in \text{COM};\ r \in \text{REG})$$

where $P^{fac}_{\_j\,(f,r)}$ is the undefined price of factor *f* (*lab*, *cap*) in region *r*.

5. Global investment of regional savings (22-26)

(22) Expected rates of return to investment in region *r*

$$R^{e}_{(r,s)} = f\,(P^{tinc}_{(\text{"}cap\text{"},s)},\ P^{t}_{\_is(\text{"}inv\text{"},s)},\ X^{cap}_{(s)\ t+1},\ \bar{X}^{fac}_{(\text{"}cap\text{"},s)}) \qquad (r \in \text{REG})$$

(23) Capital stock to be used in the next period

$$X^{cap}_{(r)\ t+1} = \bar{X}^{fac}_{(\text{"}cap\text{"},r)}\,(1 - r^{dep}_{(r)}) + Q_{\_is(\text{"}inv\text{"},r)} \qquad (r \in \text{REG})$$

where $r^{dep}_{(r)}$ is the rate of capital depreciation.

(24) CET supply of gross investment from region *r* to *s*, denominated in region *r*'s currency

$$Y^{inv}_{(r,s)} = CET\,(Y_{(\text{"}inv\text{"},r)},\ R^{e}_{\_s(r)},\ R^{e}_{(r,s)}) \qquad (r,s \in \text{REG})$$

where $R^{e}_{\_s(r)}$ is a CET index for the expected rate of return for region *r*,

$$R^{e}_{\_s(r)} = CET\,(R^{e}_{(r,s_1)},\ \ldots,\ R^{e}_{(r,s_n)}) \qquad (r \in \text{REG})$$

(25) Real investment from region *r* to *s*

$$Q^{inv}_{(r,s)} = \frac{Y^{inv}_{(r,s)}\ r^{ex}_{(r,s)}}{P^{t}_{\_is(\text{"}inv\text{"},s)}} \qquad (r,s \in \text{REG})$$

(26) Market equilibrium for real investment in region *r* (=>$Y^{NFI}_{(r)}$)

$$Q_{\_is(\text{"}inv\text{"},r)} = \sum_s Q^{inv}_{(s,r)} \qquad (r \in \text{REG})$$

6. Bilateral exchange rates (27)

(27) Nominal exchange rate: value of domestic (*s*) currency per unit of foreign (*r*) currency

$$r^{ex}_{(r,s)} = \frac{C_{(s)}}{C_{(r)}} \qquad (r,s \in \text{REG})$$

where $C_{(r)}$ is the value of domestic currency for region *r*.

Table B.2 **Sets used in the model equation system and database**

Sets	*Definitions*
COM(1,…,*m*):	Commodities (indexed by *i* for commodity or *j* for industry)
REG(1,…,*n*):	Regions (indexed by *r* for home or *s* for host)
USR(COM,*hou*,*gov*,*inv*):	Users of commodities (indexed by *u*)
SRC(*dom*,*imp*):	Sources of commodities (indexed by *s*)
FAC(*lab*,*cap*,*land*):	Factors of production (indexed by *f*)
NCF(*lab*,*land*):	Non-capital factors (indexed by *l*)
MCOM(1,…,*h*):	Margin commodities (indexed by *m*)
NCOM(1,…,*k*):	Non-margin commodities (=COM–MCOM) (indexed by *i*)