A Methods for estimating industry contributions

This appendix sets out the methods used to calculate industry contributions. The methods are based on the ‘chained Tornqvist aggregation’ approach developed in Parham (2012).

## A.1 Contributions to growth in market sector income

First, assume that the market sector income (*YMS*) is related to the income in individual industries (*Yi*) according to a Tornqvist aggregation function. In period *t*:

where the exponents are the Tornqvist weights:

and is the share of industry *i* in market sector income in year *t*.

Growth over one year can be written as:

where the hat ‘^’ over a variable signifies the growth in that variable and the ‘1’ in the superscript refers to growth over 1 year.

The growth in market sector income over *n* years accumulates the growth in each of the intervening years:

This ‘chaining’ method minimises approximation errors that arise from using base-period or end-period shares.

The contribution of industry *m* to market sector income growth over *n* years is then:

## A.2 Factor proportions and relative rewards

The objective is to analyse industry contributions to changes at the market sector level in the capital-labour ratio and the reward ratio.

Care is needed in how this is done. To base the analysis on, for example, a weighted aggregation of changes in capital-labour ratios at the industry level would give misleading results in this case. It turns out that the change in the capital-labour ratio in Mining has been negative. While capital has obviously grown strongly in Mining, labour has grown even more — but off a low base, because Mining is capital intensive.

To circumvent this problem, the contribution of industries to market sector capital deepening must be analysed as separate contributions to growth in market sector capital and to growth in market sector labour.

Growth in the market sector capital labour ratio over *n* years from year *t* can be written as:

Industry contributions to growth in market sector capital can be written as:

where the are Tornqvist weights formed from the average of the shares of industry *i* in market sector capital income in year *t+j* and the previous year.

A similar relationship can be written for the industry contributions to growth in market sector labour use:

where the are weights formed from the average of the shares of industry *i* in market sector hours worked in year *t+j* and the previous year.

Because the industry contributions are additive, the contribution of industry *i* to growth in the market sector capital-labour ratio can be written as:

The same procedure can be used to analyse industry contributions to the reward ratio. That is, they are measured as the difference between an industry’s contribution to growth in the market sector average wage and that industry’s contribution to growth in the market sector gross profit rate.

## A.3 Productivity and costs

Growth in the market sector real unit labour costs (RULC) can be decomposed into industry contributions by calculating industry decompositions of the market sector real product wage (RPW) and labour productivity (LP).

Using a subscript *MS* to represent the market sector and a superscript *t* to represent time (years):

(1)

where *NW* refers to the nominal rate of compensation, *PY* to output prices, *H* to hours worked and *Y* refers to the volume of output.

Now, each of the variables on the right hand side can be expressed as a Tornqvist aggregation of its industry counterparts. For example:

and (2)

where the subscript *i* refers to industry and *lci* is the share that industry *i* has in total (nominal) labour costs in the market sector.

The weights (*qi*in the above equation) used in the other aggregations are:

* industry shares of nominal output for *PY*,
* hours worked shares for *H* and
* nominal output shares for *Y*.

The accuracy of this method in adding up to market sector growth rates is generally high, but does vary from case to case.

Expressing (2) in terms of growth rates over a single year between t-1 and t as:

From equation (1), growth in RULC can then be expressed as:

(3)

where:

* is the average of industry *i*’s output share over *t* and *t-1* and
* is the average of industry *i*’s hours worked share over *t* and *t-1*.

Equation (3) expresses the growth in RULC in terms that are additive across industries and components (*NW*, *PY*, *H* and *Y*). Consequently, the contribution of (say) manufacturing to growth in RULC can be calculated by summing the four terms on the right hand side of (3) for *i*=manufacturing.

Growth rates over a number of years are needed for the decadal comparisons in this paper. The ‘chaining’ method is used for this purpose.

To illustrate for the case of hours worked, the average annual growth rate over *n* years from year *t* would be:

This formulation can be used on each of the terms on the right hand side of (3) to provide all components of the industry contributions market sector growth in RULC.

## A.4 Shift-share analysis

Changes in the market sector LIS can be decomposed into a component that captures changes in industry LISs and a component that captures structural change (or change in an industry’s relative size).

The market sector labour income share in year *t* () is the ratio of payments to labour () to output () in current prices.

where is the current price share of industry *i* in market sector output in year *t*.

That is, the aggregate LIS is the weighted sum of industry LISs, where the weights are the industry shares in aggregate value added.

Now, the change in the market sector labour income share over *n* years is given by:

(1)

The first term on the right hand side is the change in industry LIS component — the sum of contributions from changes in industry LISs, holding their size constant at their beginning-period share of market sector value added. The second term is the structural change or change in industry size component — the sum of contributions from the changes in the relative sizes of industries, holding their LISs constant at their starting period values.

This decomposition can be further modified to highlight the effects of structural shifts toward or away from industries with different labour intensities. To do this, industry LISs are measured relative to the market sector average. The LIS is greater than average in labour-intensive industries, whereas it is below-average in capital-intensive industries.

Adding and subtracting has no net effect on equation (1) because the sum of the output shares is unity irrespective of time. Therefore:

(2)

With this formulation, a shift toward ( is positive) capital-intensive industries ( is negative) will reduce the market sector LIS, whereas a shift toward labour-intensive industries ( is positive) will increase it.