Linking Regional Models in WALRAS

by

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1. Introduction

This memorandum describes the linking of the regional models in the version of WALRAS implemented by the Industries Assistance Commission (IAC).

WALRAS is an international general equilibrium model, developed by the Secretariat of the Organisation for Economic Co-operation and Development (OECD). The model was designed to provide estimates of the effects of agricultural policies in the major agricultural trading regions of the OECD. The model contains six regional models, representing the economies of Australia, Canada, the European Economic Community, Japan, New Zealand and the United States of America. The model is documented in OECD (1988).

The IAC has implemented the Australian regional model of WALRAS, following closely the theoretical structure described in OECD (1988), and using data supplied by the OECD. The IAC version of the Australian model is documented in McDougall and Sugden (1988). It is proposed to implement the other five regional models along similar lines.

In the version of WALRAS described in OECD (1988), no links exist between the regional models. The OECD proposes to link the regional models by adding equations relating each regions' exports and export prices to the other regions' imports and import prices. This memorandum describes the inter-regional links planned for the IAC version of WALRAS.

2. Choice of Specification

2.1 Import Demands

Import demands are modelled on the assumption that commodities imported from different regions are not perfectly substitutable for each other. Imports from different sources are combined in a constant-elasticity-of-substitution (CES) function to yield a composite imported variety of each commodity. The imported variety is then combined with a domestically produced variety to yield effective usage of each

commodity. This specification has been used by Whalley (1985), and is the specification suggested for WALRAS in OECD (1988), para 39. The specification is adopted in order to accommodate the simultaneous import and export by many countries of many commodities (the phenomenon of `cross-hauling').

This general approach is taken for two categories of demand, namely intermediate usage and consumption. These are the categories in which WALRAS already recognizes substitutability between imported and domestically produced varieties of each commodity. For other demand categories - investment, government usage, and change in stocks - no substitution between imported and domestically produced varieties is recognized. For these categories, the further assumption is now made that there is no substitution between varieties imported from different regions.

The disaggregation of imports by region of origin makes it desirable to amend one other feature of the consumer demand system. At present the model distinguishes thirteen commodities in production and in most categories of demand. These commodities are referred to below as producer commodities. In consumption, however, the model distinguishes ten commodities, referred to below as consumer commodities. Each consumer commodity is differentiated into a domestically produced and an imported variety. The domestically produced varieties of the consumer commodities are composed in fixed proportions of the domestically produced varieties of the producer commodities. Imported consumer commodities are likewise composed in fixed proportions of imported producer commodities.

This specification tends to result in low own-price elasticities of consumption demand for imported producer commodities. It also tends to result in complementarity in consumption between imported producer commodities combined in the same imported consumer commodities. Especially when imports are differentiated by source, this may lead to implausible simulation results.

For example, most imports into Australia of the producer good `dairy products' are consumed as part of the imported consumer good 'food and

non-alcoholic beverages'. Most of this good, however, consists of imports of the producer commodity `other food products'. So a change in the price of imports of `dairy products' has little effect on the price of imported `food and non alcoholic beverages', and therefore little effect on the demand for imports of `dairy products'. But a change in the price of imports of `other food products' has a much larger effect on the demand for imports of dairy products.

To avoid such implausible results, the approach taken here is to model substitution between commodities from different sources at the producer commodity level, rather than at the consumer commodity level. So in the linked version of WALRAS, consumer commodities are not differentiated by source. Consumer commodities are composed in fixed proportions of producer commodities. Consumption usage of each producer commodity is a CES function of consumption usage of the domestically produced and the composite imported varieties. Consumption usage of the composite imported variety is a CES function of consumption usage of imports from each source.

2.2 Export Supplies

Export supplies are modelled on the assumption that domestically absorbed and exported commodities are perfectly transformable in production. This is contrary to the treatment suggested in OECD (1988) para. 39, in which each industry produces a domestically absorbed and an exportable commodity subject to a constant-elasticity-of-transformation (CET) production function. The perfect transformability approach is adopted in this memorandum, as being more suitable to a long run model, in which factors of production are perfectly mobile between industries.

The imperfect transformability assumption was proposed by Dervis, de Melo and Robinson (1982) in the context of a short run model as a means of overcoming certain difficulties arising from the heterogeneity of the commodities distinguished in that model. Each commodity in the model represents a diverse class of commodities in the real economy; and likewise with industries. The share of exports in total sales may differ between the different real commodities corresponding to a single model commodity. If the different real commodities are produced by

different real industries, then fixity of factors constrains the extent to which the composition of the model commodity in terms of real commodities can be altered over the short run. For example, if export prices rise, not all types of factors can be reallocated over the short run from domestically oriented to export oriented real industries. So the elasticity of supply of exports is lower than would be indicated by the model. Furthermore, the rise in the prices of exportable real commodities does not cause the prices of domestically absorbed real commodities to rise equally. So domestic prices are not coupled rigidly to export prices, as the model would indicate.

To overcome these problems Dervis, de Melo and Robinson (1982) differentiate each commodity in the model into a domestically absorbed and an exported variety, produced by the same industry under conditions of imperfect transformation. This has the desired effects of reducing the elasticity of supply of exports, and partially decoupling domestic prices from export prices. These are also the advantages claimed for this approach by OECD (1988).

The considerations which led Dervis, de Melo and Robinson (1982) to adopt this approach rest on the assumption that capital is immobile between industries. If all factors are mobile, then there is no constraint on changes in the composition of model commodities in terms of real commodities. Furthermore, export prices and domestic prices are likely to be tightly coupled. The reason is that both domestically oriented and export oriented real industries face the same input prices (including rental prices for capital). So domestically oriented and export oriented industries are likely to incur similar changes in unit costs, and to make similar changes to their output prices.

In WALRAS, all factors of production are treated as mobile between industries. The model therefore takes a long run perspective, in which the considerations which support the imperfect transformability approach do not apply. This memorandum adopts accordingly the perfect transformability approach. In doing so, it increases the risk that the composition of output in the individual regions in the model may be implausibly insensitive with respect to policy shocks.

2.3 The Treatment of the Rest of the World

The six regions represented in WALRAS constitute a large part, but by no means all, of the world economy. Besides linking the regional models together, it is necessary to take into account trade between the six formally modelled regions and the rest of the world.

In this memorandum all import demands and export supplies by the rest of the world are treated as completely inelastic. This is a drastic simplification, more drastic than that commonly employed in single country general equilibrium models. In those models the supply of exports by the rest of the world is commonly assumed to be perfectly elastic, while the demand by the rest of the world for imports of each commodity is assigned a constant finite elasticity. That treatment is suitable if the economy of the rest of the world is large with respect to that of the formally modelled region. In that case, economic changes in the formally modelled region have no considerable effect on the level and composition of output in the rest of the world. In the contrary case, when the modelled region is large enough to influence output in the rest of the world, it is no longer reasonable to assume that demand by the rest of the world for imports of a commodity from the modelled region depends only on the price of the imported commodity. Rather, it will depend on all import and export prices. Similar remarks apply to exports from the rest of the world.

It appears therefore that in WALRAS, in which the rest of the world is not large relative to the group of modelled regions, no simple specification of its import demands and export supplies is suitable. To provide a satisfactory account of those variables, it would be necessary to incorporate the rest of the world into the model as one or a number of regions. That task is well outside the scope of the present memorandum. This memorandum accordingly refrains from offering any explanation of changes in rest-of-the-world imports and exports. Those variables are therefore exogenous in the linked model. It must then be acknowledged that the linked model excludes certain adjustment processes, namely all those processes which are mediated by the rest of the world.

3. Structure of the Inter-Regional Linking Extension to WALRAS

3.1 Introduction

The regional models in WALRAS are linked by the addition of adding equations, variables and parameters. The model to which these additions are made is an assemblage of six regional models, each having the same structure as the IAC version of the WALRAS model of Australia, documented in McDougall and Sugden (1988).

Tables 1, 2 and 3 list the equations, variables and parameters added to the non-linear version of WALRAS. Tables 4 and 5 list the corresponding linearized equations and their parameters. The notation follows that of McDougall and Sugden (1988), except that regional superscripts are added to the variables and parameters of the regional models. Regional superscripts range from 1 to 6, and indicate regions as shown in Table 6. Variables representing quantities or prices of imports, or quantities of exports, also bear subscripts indicating the source of the imports or the destination of the exports. Subscripts from 1 to 6 indicate regions represented in the model, as shown in Table 6; a subscript of 7 indicates the rest of the world.

3.2 Demands for Inputs into Current Production

Equation 3.1 of Table 1 expresses usage of imported inputs from each source into current production in each industry as the product of an input-output coefficient and the level of output in the industry.

Usage in each industry of the composite imported variety of each commodity is assumed to be a linearly homogeneous CES function of usage of the variety imported from each source. Using input-output coefficients, the relation may be written

$$A^{Ir}_{ij} = \left(\sum_s \gamma^{Ir}_{(is)j} \left(A^{Ir}_{(is)j}\right)^{(\eta^{Ir}_i - 1)/\eta^{Ir}_i}\right)^{\eta^{Ir}_i/(\eta^{Ir}_i - 1)},$$

$$i = 1,\ldots,n_S \quad j = 1,\ldots,n_S, \quad r = 1,\ldots,n_r.$$

Here n_s denotes that number of industries or producer commodities in the model, and n_r denotes the number of regions (not including the 'rest of the world'). In the model as developed by the OECD, n_s is equal to thirteen, and n_r to six.

The elasticity of substitution between variables imported from different sources is assumed to be the same in all industries, but may vary across commodities and regions. Solution of the cost minimization problem yields equations (3.3) for the input-output coefficients, and equation (3.2) for the price of the composite imported variety.

3.3 Investment Demand

Formation of the capital good in each region in WALRAS is Leontieff in the domestically produced variety and each imported variety of each commodity. Equations (3.4) of Table 1 are the investment demand equations for the imported varieties. They replace equations (2.15) of McDougall and Sugden (1988).

3.4 Consumer Demand

As stated in Section 2.1, the consumer demand system in the regional models is redesigned for the linked model. In the linked model, the consumer commodities are not differentiated into domestically produced and imported varieties. Consumption of each consumer commodity is Leontieff in each producer commodity, as shown in equation (3.6). Consumption of each producer commodity by way of each consumer commodity is CES in the domestically produced and the composite imported varieties of the producer commodity,

$$PC^r_{ig} = \left(\omega^{Dr}_{ig} \left(PC^{Dr}_{ig}\right)^{(\beta^r_g-1)/\beta^r_g} + \omega^{Ir}_{ig} \left(PC^{Ir}_{ig}\right)^{(\beta^r_g-1)/\beta^r_g}\right)^{\beta^r_g/(\beta^r_g-1)},$$

$$i = 1,\ldots,n_s,$$

$$g = 1,\ldots,n_c,$$

$$r = 1,\ldots,n_r.$$

Here n_c denotes the number of consumer commodities. In the sectoral classification adopted by the OECD, n_c is equal to ten.

The elasticity of substitution is allowed to vary across consumer commodities, but not across producer commodities. Utility maximization yields equations (3.8) and (3.9) for consumption of the domestically produced and composite imported varieties of each producer commodity by way of each consumer commodity. Furthermore, equation (3.7) is obtained for the price of the i'th producer commodity when consumed by way of the g'th consumer commodity. Equation (3.5) expresses consumer commodity prices in terms of producer commodity prices.

Consumption of the composite imported variety of each producer commodity by way of each consumer commodity is CES in the varieties imported from each source,

$$PC^{Ir}_{ig} = \left(\sum_{s} \omega^{Ir}_{(is)g} \left(PC^{Ir}_{(is)g}\right)^{(\beta^{Ir}_{g}-1)/\beta^{Ir}_{g}}\right)^{\beta^{Ir}_{g}/(\beta^{Ir}_{g}-1)}, \qquad \begin{aligned} i &= 1,\ldots,n_s, \\ g &= 1,\ldots,n_c, \\ r &= 1,\ldots,n_r. \end{aligned}$$

Again the elasticity of substitution is allowed to vary across consumer commodities, but not across producer commodities. Utility maximization yields equations (3.11) for consumption of each imported variety of each producer commodity by way of each consumer commodity. Furthermore, equation (3.10) is obtained for the price of the composite imported variety of each producer commodity consumed by way of each consumer commodity. Finally equations (3.12) and (3.13) express total consumption in each region of the domestically produced and each imported variety as the sum of consumption by way of each consumer commodity.

3.5 Export Demand

Equations (3.14) equate exports of each commodity from each region to each destination with imports to that destination from that region. A conversion factor is required to convert quantities traded from the units used for the importing countries to the units used for the exporting country. Equations (3.15) express total exports of each

commodity from each region as the sum of exports to each source. These equations are applied not only to exports from regions represented in the model, but also to exports from the rest of the world.

3.6 Government Demand

Government usage of each producer commodity is Leontieff in the domestically produced and each imported variety. Equations (3.17) are the government demand equations for the imported varieties. Equation (3.16), which replaces equation (2.27) of McDougall and Sugden (1988), defines a price index for government usage of producer commodities.

3.7 Change in Stocks

The composition by source of change in stocks of each commodity is assumed to be constant. Equations (3.18), which replace equations (2.31) of McDougall and Sugden (1988), are the inventory investment demand equations for the imported varieties.

3.8 Demand for Imports by the Rest of the World

The equations described in the previous sections explain demands for imports by regions represented in the model by reference to the structure of production and absorption in these regions. A simpler method must be used to explain imports by the rest of the world. That simpler method is explained in this section.

Demand by the rest of the world for imports depends on the price of imports relative to prices in the rest of the world. The elasticity of demand with respect to relative price is constant. For the six region model, in which the rest of the world is not large in comparison to the explicitly represented regions as a whole, it is envisaged that the elasticity will be set equal to zero. Thus demand by the rest of the world for imports will be completely inelastic, as stated in Section 2.3. But for particular applications of the model, the model may be modified so that the number of regions is less than six. In some of these applications, the rest of the world may be large relative to

the explicitly represented regions as a whole. The structure of the model allows users to specify positive import demand elasticities by the rest of the world in these applications.

Even when total demand by the rest of the world for imports of each commodity is held constant, demand for imports from individual regions may vary. Imports from each region are combined in CES functions to form a composite imported variety of each commodity:

$$IMP_i^{n_r+1} = \left[\sum_s \omega_{(is)}^{ROW} \left(P_{W(is)}^{n_r+1}\right)^{(\beta_i^{ROW}-1)/\beta_i^{ROW}}\right]^{\beta_i^{ROW}/(\beta_i^{ROW}-1)} .$$

Equation (3.19) expresses demand by the rest of the world for the composite imported variety of each commodity in terms of the price of the composite imported variety relative to the domestically produced variety. Equation (3.20) defines a price index for the composite imported variety in terms of the prices of the varieties imported from each country. Equation (3.21) shows how demand by the rest of the world for imports is allocated between exporting regions, on the basis of the price of exports from each region.

3.9 The Price System

Equations (3.23), which replace equations (2.33) of McDougall and Sugden (1988), express the basic price of each imported variety of each commodity in terms of the landed duty-free price, the exchange rate, and the rate of duty on the variety. Equations (3.22) express the landed duty free price in the importing country in terms of the free on board price in the exporting country, and the price of an international trade and transport margin commodity.

Equations (3.24), which replace equations (2.35) of McDougall and Sugden (1988), express the price paid by the j'th industry for each imported variety of each commodity in terms of the basic price of the imported variety and a commodity tax rate. The same commodity tax rate is assumed to apply to imports from all sources.

Equations (3.25) and (3.26), which correspond roughly to equations (2.36) and (2.37) of McDougall and Sugden (1988), express the price in consumption usage of the domestically produced variety and each imported variety of each commodity, in terms of the basic price of the variety and a commodity tax rate. Equations (3.25) and (3.26) do not correspond exactly to equations (2.36) and (2.37) of the previous memorandum, in that they calculate prices for producer commodities, rather than consumer commodities.

Equation (3.27) expresses the price of the international trade and transport commodity as a weighted average of the price in each country of the last commodity of the producer commodity classification ('other private services').

3.10 The Market-Clearing Equations

Equations (3.28), which replace equations (2.40) of McDougall and Sugden (1988), are identities expressing total imports of each commodity into each region by summing over demand categories.

3.11 Miscellaneous Equations and Macro Identities

Equations (3.29), which replace equation (2.45) of McDougall and Sugden (1988), define the purchase price of capital in each region. Equations (3.30) and (3.31), which replace equations (2.46) and (2.47) of the earlier memorandum, define aggregate consumption expenditure and the consumer price index Equations (3.32) and (3.33) define aggregate inventory investment expenditure and a price index for inventory investment. Equations (3.34) and (3.35) define aggregate imports and an import price index. Finally equation (3.36) defines aggregate government revenue.

4. The Linearised Inter-Regional Linking Extension

In implementing the WALRAS model, the IAC has linearised the equation system. So the inter-regional linking extension must also be

linearised. Table 4 presents the equation system of Table 1 in percentage change form. Parameters unique to the linearised equation system are listed in Table 5.

Most of the variables in the linearised system are percentage changes in variables in the non-linear system. The exceptions are the tax rate variables. In the linearised system these are represented by percentage changes in the power of the tax, rather than percentage changes in the tax rate. The power of a tax is defined as one plus the tax rate (measured as a fraction). For example, if a tax rate is 49 per cent, then the power of the tax is equal to 1.49.

5. Model Closure

Closure of the linked version of WALRAS is generally similar to closure of the unlinked version. The main differences are as follows.

- In the unlinked version, the variables P^r_{Wi}, representing world prices, are typically exogenous; but in the linked version, the corresponding variables $P^r_{W(is)}$, representing LDF foreign currency prices, are typically endogenous.

- In the unlinked version, all of the variables P^r_{Ei}, representing FOB export prices, are typically endogenous; but in the linked version, the variables P^r_{Ei}, representing FOB prices of exports from the rest of the world, are exogenous.

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TABLE 1 : THE EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS

Identifier	Equation	Subscript range	Number	Description	Corresponding equation in single region model[a]
(3.1)	$X^{Ir}_{(is)j} = A^{Ir}_{(is)j} Q^r_j$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r (n_r+1)$	Demand by the j'th industry in the r'th region for the variety of the i'th commodity imported from the s'th region.	None.
(3.2)	$PT^{Ir}_{ij} = \left[\sum_s (\gamma^{Ir}_{(is)j})^{n_i^{Ir}} (PT^{Ir}_{(is)j})^{1-n_i^{Ir}}\right]^{1/(1-n_i^{Ir})}$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r$	Price index for the composite imported variety of the i'th commodity used by the j'th industry in the r'th region.	None.
(3.3)	$A^{Ir}_{(is)j} = (\gamma^{Ir}_{(is)j})^{n_i^{Ir}} (PT^{Ir}_{(is)j})^{-n_i^{Ir}} \left[\sum_s (\gamma^{Ir}_{(is)j})^{n_i^{Ir}} (PT^{Ir}_{(is)j})^{1-n_i^{Ir}}\right]^{n_i^{Ir}/(1-n_i^{Ir})} A^{Ir}_{ij}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Input-output coefficient for usage by the j'th industry in the r'th region of the variety of the i'th commodity imported from the s'th region.	None.
(3.4)	$INV^{Ir}_{(is)} = \Gamma^{Ir}_{(is)} INV^{Tr}_R$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Investment usage in the r'th region of the variety of the i'th commodity imported from the s'th region.	(2.15)
(3.5)	$P^r_{Cg} = \sum_i \xi^r_{ig} P^r_{Cig}$	$g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_r$	Price of the g'th consumer commodity in the r'th region.	Equations (3.5) - (3.9) and (3.12) - (3.13) correspond together to equations (2.18) - (2.22) of the single region model.
(3.6)	$PC^r_{ig} = \xi^r_{ig} C^r_g$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_s n_c n_r$	Consumption in the r'th region of the i'th producer commodity by way of the g'th consumer commodity.	See above.
(3.7)	$P^r_{Cig} = \left[(\omega^{Dr}_{ig})^{\beta^r_g} (P^{Dr}_{Ci})^{1-\beta^r_g} + (\omega^{Ir}_{ig})^{\beta^r_g} (P^{Ir}_{Cig})^{1-\beta^r_g}\right]^{1/(1-\beta^r_g)}$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_s n_c n_r$	Price index for the i'th producer commodity when consumed in the r'th region by way of the g'th consumer commodity.	See above.

(Continued)

TABLE 1 : THE EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Identifier	Equation	Subscript range	Number	Description	Corresponding equation in single region model[a]
(3.8)	$PC^{Dr}_{ig} = (\omega^{Dr}_{ig})^{\beta^r_g} (P^{Dr}_{Ci})^{-\beta^r_g}$ $[(\omega^{Dr}_{ig})^{\beta^r_g} (P^{Dr}_{Ci})^{1-\beta^r_g} + (\omega^{Ir}_{ig})^{\beta^r_g} (P^{Ir}_{Cig})^{1-\beta^r_g}]^{\beta^r_g/(1-\beta^r_g)}$ PC^r_{ig}	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region of the domestically produced variety of the i'th producer commodity, by way of the g'th consumer commodity.	See above.
(3.9)	$PC^{Ir}_{ig} = (\omega^{Ir}_{ig})^{\beta^r_g} (P^{Ir}_{Cig})^{-\beta^r_g}$ $[(\omega^{Dr}_{ig})^{\beta^r_g} (P^{Dr}_{Ci})^{1-\beta^r_g} + (\omega^{Ir}_{ig})^{\beta^r_g} (P^{Ir}_{Cig})^{1-\beta^r_g}]^{\beta^r_g/(1-\beta^r_g)}$ PC^r_{ig}	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region of the composite imported variety of the i'th producer commodity by way of the g'th consumer commodity.	See above.
(3.10)	$P^{Ir}_{Cig} = [\sum_s (\omega^{Ir}_{(is)g})^{\beta^{Ir}_g} (P^{Ir}_{C(is)})^{1-\beta^{Ir}_g}]^{1/(1-\beta^{Ir}_g)}$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Price index for the composite imported variety of the i'th producer commodity consumed by way of the g'th consumer commodity in the r'th region.	None.
(3.11)	$PC^{Ir}_{(is)g} = (\omega^{Ir}_{(is)g})^{\beta^{Ir}_g} (P^{Ir}_{C(is)})^{-\beta^{Ir}_g}$ $[\sum_s (\omega^{Ir}_{(is)g})^{\beta^{Ir}_g} (P^{Ir}_{C(is)})^{1-\beta^{Ir}_g}]^{\beta^{Ir}_g/(1-\beta^{Ir}_g)}$ PC^{Ir}_{ig}	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r (n_r+1)$	Consumption in the r'th region, by way of the g'th consumer commodity, of the variety of the i'th producer commodity imported from the s'th region.	None.
(3.12)	$PC^{Dr}_i = \sum_g PC^{Dr}_{ig}$	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r$	Consumption in the r'th region of the domestically produced variety of the i'th producer commodity.	See entry for equation (3.9).

(Continued)

Identifier	Equation	Subscript range	Number	Description	Corresponding equation in single region model[a]
(3.13)	$PC^{Ir}_{(is)} = \sum_{g} PC^{Ir}_{ig}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Consumption in the r'th region of the variety of the i'th producer commodity imported from the s'th region.	See entry for equation (3.9).
(3.14)	$E^{r}_{si} = IMP^{s}_{(ir)}/TR^{s}_{(ir)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s (n_r+1)^2$	Exports of the i'th commodity from the r'th region to the s'th region.	None.
(3.15)	$E^{r}_{i} = \sum_{s} E^{r}_{si}$	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s (n_r+1)$	Exports of the i'th commodity from the r'th region.	None.
(3.16)	$P^{r}_{G} = \sum_{i} \chi^{Dr}_{i} (1 + T^{r}_{Gi}) P^{Dr}_{i} + \sum_{i} \sum_{s} \chi^{Ir}_{(is)} (1 + T^{r}_{Gi}) P^{Ir}_{(is)}$	$r = 1,\ldots,n_r$.	n_r	Price index for government usage of producer commodities.	(2.27).
(3.17)	$GOV^{Ir}_{(is)} = \chi^{Ir}_{(is)} G^{r}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Government usage in the r'th region of the variety of the i'th commodity imported from the s'th region.	(2.29)
(3.18)	$DSTOC^{Ir}_{(is)} = \Omega^{Ir}_{(is)} DSTOC^{Tr}_{R}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Change in stocks in the r'th region of the variety of the i'th commodity imported from the s'th region.	(2.31).
(3.19)	$IMP^{n_r+1}_{i} = \delta_i (P^{n_r+1}_{WI}/P^{n_r+1}_{Ei})^{\epsilon_i}$	$i = 1,\ldots,n_s$.	n_s	Demand by the rest of the world for the composite imported variety of the i'th commodity.	(2.23).

(Continued)

TABLE 1 : THE EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Identifier	Equation	Subscript range	Number	Description	Corresponding equation in single region model[a]
(3.20)	$P_{Wi}^{n_r+1} = \left[\sum_s (\omega_{(is)}^{ROW})^{\beta_i^{ROW}} (P_{W(is)}^{n_r+1})^{1-\beta_i^{ROW}}\right]^{1/(1-\beta_i^{ROW})}$	$i = 1,\ldots,n_s$.	n_s	Price index for the composite variety of the i'th commodity imported into the rest of the world.	None.
(3.21)	$IMP_{(is)}^{n_r+1} = (\omega_{(is)}^{ROW})^{\beta_i^{ROW}} (P_{W(is)}^{n_r+1})^{-\beta_i^{ROW}}$ $\left[\sum_s (\omega_{(is)}^{I,n+1})^{\beta_i^{ROW}} (P_{W(is)}^{n_r+1})^{1-\beta_i^{ROW}}\right]^{\beta_i^{ROW}/(1-\beta_i^{ROW})}$ $IMP_i^{n_r+1}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$.	$n_s(n_r+1)$	Demand by the rest of the world for the variety of the i'th commodity imported from the s'th region.	None.
(3.22)	$P_{W(is)}^r = A_{B(is)}^r P_{Ei}^s/ER^s + A_{M(is)}^r P_T$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)^2$	Foreign currency LDF price in the r'th region of the variety of the i'th commodity imported from the s'th region.	None.
(3.23)	$P_{(is)}^{Ir} = (1 + D_{(is)}^r)\, ER^r \cdot P_{W(is)}^r$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Basic price in the r'th region of the variety of the i'th commodity imported from the s'th region.	(2.33)
(3.24)	$PT_{(is)j}^{Ir} = (1 + T_{ij}^{Ir})P_{(is)}^{Ir}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r (n_r+1)$	Price paid by the j'th industry in the r'th region for the variety of the i'th commodity imported from the s'th region.	(2.35)
(3.25)	$P_{Ci}^{Dr} = (1 + T_{Ci}^{Dr})P_i^{Dr}$	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r$	Price paid by households in the r'th region for the domestically produced variety of the i'th producer commodity.	(2.36)

(Continued)

TABLE 1 : THE EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Identifier	Equation	Subscript range	Number	Description	Corresponding equation in single region model[a]
(3.26)	$P^{Ir}_{C(is)} = (1 + T^{Ir}_{Ci})\, P^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Price paid by households in the r'th region for the variety of the i'th producer commodity imported from the s'th region.	(2.37)
(3.27)	$P_T = \sum_r A_{Mr}\, P^r_{E,n_s} / ER^r$		1	Price of international freight.	None.
(3.28)	$IMP^r_{(is)} = \sum_j X^{Ir}_{(is)j} + INV^{Ir}_{(is)} + PC^{Ir}_{(is)} + GOV^{Ir}_{(is)} + DSTOC^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Imports of the i'th commodity from the s'th region into the r'th region.	(2.40)
(3.29)	$P^r_I = [\sum_i \Gamma^{Dr}_i (1 + T^r_{Ii}) P^{Dr}_i + \sum_i \sum_s \Gamma^{Ir}_{(is)} (1 + T^r_{Ii}) P^{Ir}_{(is)}]/RR$	$r = 1,\ldots,n_r$.	n_r	Purchase price of capital in the r'th region.	(2.45).
(3.30)	$C^{Tr} = \sum_i P^{Dr}_{Ci}\, PC^{Dr}_i + \sum_i \sum_s P^{Ir}_{C(is)} PC^{Ir}_{(is)}$	$r = 1,\ldots,n_r$.	n_r	Aggregate consumption expenditure in the r'th region.	(2.46).
(3.31)	$CPI^r = \sum_i A^{Dr}_{Ci}\, P^{Dr}_{Ci} + \sum_i \sum_s A^{Ir}_{C(is)}\, P^{Ir}_{C(is)}$	$r = 1,\ldots,n_r$.	n_r	Consumer price index in the r'th region.	(2.47)
(3.32)	$DSTOC^{Tr} = \sum_i (1 + T^r_{Si})\, P^{Dr}_i\, DSTOC^{Dr}_i + \sum_i \sum_s (1 + T^r_{Si})\, P^{Ir}_{(Si)}\, DSTOC^{Ir}_{(is)}$	$r = 1,\ldots,n_r$.	n_r.	Aggregate change in stocks in the r'th region at current prices.	(2.52).
(3.33)	$DSTOCPI^r = \sum_i \Omega^{Dr}_i (1 + T^r_{Si}) P^{Dr}_i + \sum_i \sum_s \Omega^{Ir}_{(is)} (1 + T^r_{Si}) P^{Ir}_{(is)}$	$r = 1,\ldots,n_r$.	n_r	Price index for change in stocks in the r'th region.	(2.53)

(Continued)

TABLE 1 : THE EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Identifier	Equation	Subscript range	Number	Description	Corresponding equation in single region model[a]
(3.34)	$IMP^{Tr} = \sum_i \sum_s P^r_{W(is)} \, IMP^r_{(is)}$	$r = 1,\ldots,n_r$.	n_r	Aggregate imports into the r'th region at foreign currency LDF prices.	(2.54).
(3.35)	$IMPPI^r = \sum_i \sum_s A^r_{I(is)} \, P^r_{W(is)}$	$r = 1,\ldots,n_r$.	n_r	Price index for imports into the r'th region.	(2.55)
(3.36)	$R^r_G = \sum_i \sum_s D^r_{is} \, ER^r \, P^r_{W(is)} \, IMP^r_{(is)} + \sum_i \sum_j T^{Dr}_{ij} \, P^{Dr}_i \, X^{Dr}_{ij}$	$r = 1,\ldots,n_r$.	n_r	Government revenue in the r'th region.	(2.64)
	$+ \sum_i \sum_j \sum_s T^{Ir}_{(is)j} \, P^{Ir}_{(is)} \, X^{Ir}_{(is)j}$				
	$+ \sum_i T^r_{Ii} P^{Dr}_i \, INV^{Dr}_i + \sum_i \sum_s T^r_{Ii} P^{Ir}_{(is)} INV^{Ir}_{(is)}$				
	$+ \sum_i T^{Dr}_{Ci} \, P^{Dr}_i \, PC^{Dr}_i + \sum_i \sum_s T^r_{Ci} \, P^{Ir}_{(is)} \, PC^{Ir}_{(is)}$				
	$+ \sum_i T^r_{Ei} P^{Dr}_i \, E^r_i$				
	$+ \sum_i T^r_{Gi} \, P^{Dr}_i \, GOV^{Dr}_i + \sum_i \sum_s T^r_{Gi} P^{Ir}_{(is)} \, GOV^{Ir}_{(is)}$				
	$+ \sum_i T^r_{Si} P^{Dr}_i DSTOC^{Dr}_i + \sum_i \sum_s T^r_{Si} \, P^{Ir}_{(is)} \, DSTOC^{Ir}_{(is)}$				
	$- \sum_j S^r_{Qj} \, P^{Dr}_j \, Q^r_j + \tau^r_y + T^r_Y \, Y^r$				

a Numbers identify equations in Table 1 of McDougall and Sugden (1988).

TABLE 2 : VARIABLES UNIQUE TO THE INTER-REGIONAL LINKING EXTENSION TO WALRAS

Variable	Subscript range	Number	Description	Deleted variable
$A^{Ir}_{(is)j}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r (n_r+1)$	Input-output coefficient for usage by the j'th industry in the r'th region of the variety of the i'th commodity imported from the s'th region.	None
$PT^{Ir}_{(is)j}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r (n_r+1)$	Price paid by the j'th industry in the r'th region for the variety of the i'th commodity imported from the s'th region.	None
$X^{Ir}_{(is)j}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r (n_r+1)$	Demand by the j'th industry in the r'th region for the variety of the i'th commodity imported from the s'th region.	None
$PC^{Ir}_{(is)g}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r (n_r+1)$	Consumption in the r'th region, by way of the g'th consumer commodity, of the variety of the i'th producer commodity imported from the s'th region.	None
P^{r}_{Cig}	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Price index for the i'th producer commodity when consumed by way of the g'th consumer commodity in the r'th region.	None
P^{Ir}_{Cig}	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Price index for the composite imported variety of the i'th producer commodity consumed by way of the g'th consumer commodity in the r'th region.	None
PC^{r}_{ig}	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region, by way of the g'th consumer commodity, of the i'th producer commodity.	None
PC^{Dr}_{ig}	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region, by way of the g'th consumer commodity, of the domestically produced variety of the i'th producer commodity.	C^{Dr}_{g}

(Continued)

TABLE 2 : VARIABLES UNIQUE TO THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Variable	Subscript range	Number	Description	Deleted variable
PC^{Ir}_{ig}	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region, by way of the g'th consumer commodity, of the composite imported variety of the i'th producer commodity.	C^{Ir}_{g}
E^{r}_{si}	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)^2$	Exports of the i'th commodity from the r'th region to the s'th region.	None
$IMP^{r}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)^2$	Imports of the i'th commodity from the s'th region into the r'th region.	IMP^{r}_{i}
$P^{r}_{W(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)^2$	Foreign currency LDF price of imports of the i'th commodity from the s'th region into the r'th region.	P^{r}_{Wi}
$PC^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r(n_r+1)$	Consumption in the r'th region of the variety of the i'th commodity imported from the s'th region.	PC^{Ir}_{i}
$D^{r}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r(n_r+1)$	Tariff on imports of the i'th commodity from the s'th region into the r'th region.	D^{r}_{i}
$DSTOC^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r(n_r+1)$	Change in stocks in the r'th region of the variety of the i'th commodity imported from the s'th region.	$DSTOC^{Ir}_{i}$
$GOV^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r(n_r+1)$	Government usage in the r'th region of the variety of the i'th commodity imported from the s'th region.	GOV^{Ir}_{i}
$INV^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r(n_r+1)$	Investment usage in the r'th region of the variety of the i'th commodity imported from the s'th region.	INV^{Ir}_{i}
$P^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r(n_r+1)$	Basic price in the region of the variety of the i'th commodity imported from the s'th region.	P^{Ir}_{i}

(Continued)

TABLE 2 : VARIABLES UNIQUE TO THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Variable	Subscript range	Number	Description	Deleted variable
$P^{Ir}_{C(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Price paid by households in the r'th region for the variety of the i'th commodity imported from the s'th region.	P^{Ir}_{Cg}
P^{r}_{Ei}	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)$	FOB price of exports of the i'th commodity from the r'th region.	P^{r}_{Ei}
P^{Dr}_{Ci}	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r$	Price paid by households in the r'th region for the domestically produced variety of the i'th commodity.	P^{Dr}_{Cg}
$IMP_i^{n_r+1}$	$i = 1,\ldots,n_s$.	n_s	Imports into the rest of the world of the composite imported variety of the i'th commodity.	None
$P_{Wi}^{n_r+1}$	$i = 1,\ldots,n_s$.	n_s	Price index for the composite variety of the i'th commodity imported into the rest of the world.	None
P_T		1	Price of international freight.	None

TABLE 3 : PARAMETERS UNIQUE TO THE INTER-REGIONAL LINKING EXTENSION TO WALRAS

Equation	Coefficient	Description	Source
(3.2)-(3.3)	$\gamma^{Ir}_{(is)j}$	Intensity of the composite imported variety of the i'th commodity used by the j'th industry in the r'th region, in the variety imported from the s'th region.	Calibration.
(3.2), (3.3)	η^{Ir}_{i}	Elasticity of substitution in intermediate usage in the r'th region between varieties of the i'th commodity imported from different regions.	Elasticities file.
(3.4)	$\Gamma^{Ir}_{(is)}$	Input-output coefficient for usage in capital creation in the r'th region of the variety of the i'th commodity imported from the s'th region.	Calibration.
(3.5), (3.6)	ξ^{r}_{ig}	Input-output coefficient in the r'th region for the i'th producer commodity and the g'th consumer commodity.	Calibration.
(3.7)-(3.9)	ω^{Dr}_{ig}	Intensity of consumption in the r'th region of the i'th producer commodity by way of the g'th consumer commodity, in the domestically produced variety of the i'th producer commodity.	Calibration.
(3.7)-(3.9)	ω^{Ir}_{ig}	Intensity of consumption in the r'th region of the i'th producer commodity by way of the g'th consumer commodity, in the composite imported variety.	Calibration.
(3.7)	β^{r}_{g}	Elasticity of substitution between the domestically produced and composite imported varieties of producer commodities, when consumed in the r'th region by way of the g'th consumer commodity.	Calibration.
(3.10), (3.11)	$\omega^{Ir}_{(is)g}$	Intensity of the composite imported variety of the i'th producer commodity consumed in the r'th region by way of the g'th consumer commodity, in the variety imported from the s'th region.	Calibration.
(3.10), (3.11)	β^{Ir}_{g}	Elasticity of substitution between varieties of a producer commodity imported from different regions, when consumed in the r'th region by way of the g'th consumer commodity.	Calibration.
(3.14)	$TR^{s}_{(ir)}$	Ratio of the LDP price of exports from the r'th region to the s'th region to the FOB price, in common currency units.	Calibration.
(3.16), (3.17)	$\chi^{Ir}_{(is)}$	Input-output coefficient for the variety of the i'th commodity imported from the s'th region, and total government spending on producer commodities in the r'th region.	Calibration.
(3.18), (3.33)	$\Omega^{Ir}_{(is)}$	Input-output coefficient for the variety of the i'th commodity imported from the s'th region and change in stocks in the r'th region.	Calibration.

(Continued)

TABLE 3 : PARAMETERS UNIQUE TO THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Cont'd)

Equation	Coefficient	Description	Source
(3.20), (3.21)	β_i^{ROW}	Elasticity of substitution between varieties of the i'th commodity imported into the rest of the world from different regions.	Elasticities file.
(3.22)	$A^r_{M(is)}$	Share of international transport margin in the LDF price of imports from the s'th region into the r'th region.	Calibration.
(3.22)	$A^r_{B(is)}$	Share of the FOB price in the LDF price of imports from the s'th region into the r'th region.	Calibration.
(3.27)	A_{Mr}	Share of the r'th region in total supply of international freight.	Calibration.
(3.31)	$A^{Ir}_{C(is)}$	Average budget share in the r'th region of the variety of the i'th commodity imported from the s'th region.	Calibration.
(3.35)	$A^r_{I(is)}$	Share of the variety of the i'th commodity imported from the s'th source in total imports into the r'th region.	Calibration.

TABLE 4 : THE LINEARISED EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS

Identifier	Equation	Subscript range[a]	Number[a]	Description
(4.1)	$x^{Ir}_{(is)j} = a^{Ir}_{(is)j} + q^{r}_{j}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r (n_r+1)$	Demand by the j'th industry in the r'th region for the variety of the i'th commodity imported from the s'th region.
(4.2)	$pt^{Ir}_{ij} = \sum_s S^{Ir}_{(is)j}\ pt^{Ir}_{(is)j}$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r$	Price index for the composite imported variety of the i'th commodity used by the j'th industry in the r'th region.
(4.3)	$a^{Ir}_{(is)j} = a^{Ir}_{ij}\ n^{Ir}_{i}\ (pt^{Ir}_{(is)j} - pt^{Ir}_{ij})$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s^2 n_r (n_r+1)$	Input-output coefficient for usage by the j'th industry in the r'th region of the variety of the i'th commodity imported from the s'th region.
(4.4)	$inv^{Ir}_{(is)} = inv^{Tr}_{R}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Investment usage in the r'th region of the variety of the i'th commodity imported from the s'th region.
(4.5)	$p^{r}_{Cg} = \sum_i S^{r}_{Cig}\ p^{r}_{Cig}$	$g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_r$	Price of the g'th consumer commodity in the r'th region.
(4.6)	$pc^{r}_{ig} = c^{r}_{g}$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region of the i'th producer commodity by way of the g'th consumer commodity.
(4.7)	$p^{r}_{Cig} = S^{Dr}_{Cig}\ p^{Dr}_{Ci} + S^{Ir}_{Cig}\ p^{Ir}_{Cig}$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Price index for the i'th producer commodity when consumed in the r'th region by way of the g'th consumer commodity.
(4.8)	$pc^{Dr}_{ig} = pc^{r}_{ig} - \beta^{r}_{g}\ (p^{Dr}_{Ci} - p^{r}_{Cig})$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region of the domestically produced variety of the i'th producer commodity, by way of the g'th consumer commodity.
(4.9)	$pc^{Ir}_{ig} = pc^{r}_{ig} - \beta^{r}_{g}\ (p^{Ir}_{Cig} - p^{r}_{Cig})$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Consumption in the r'th region of the composite imported variety of the i'th producer commodity, by way of the g'th consumer commodity.
(4.10)	$p^{Ir}_{Cig} = \sum_s S^{Ir}_{C(is)g}\ p^{Ir}_{C(is)}$	$i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$	Price index for the composite imported variety of the i'th producer commodity consumed in the r'th region by way of the g'th consumer commodity.
(4.11)	$pc^{Ir}_{(is)g} = pc^{Ir}_{ig} - \beta^{Ir}_{g}\ (p^{Ir}_{C(is)} - p^{Ir}_{Cig})$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $g = 1,\ldots,n_c$, $r = 1,\ldots,n_r$.	$n_c n_s n_r$ (n_r+1)	Consumption in the r'th region, by way of the g'th consumer commodity, of the variety of the i'th producer commodity imported from the s'th region.
(4.12)	$pc^{Dr}_{i} = \sum_g S^{Dr}_{ig}\ pc^{Dr}_{ig}$	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r$	Consumption in the r'th region of the domestically produced variety of the i'th producer commodity.

(Continued)

TABLE 4 : THE LINEARISED EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(4.13)	$pc^{Ir}_{(is)} = \sum_g S^{Ir}_{(is)g}\, pc^{Ir}_{(is)g}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Consumption in the r'th region of the variety of the i'th producer commodity imported from the s'th region.
(4.14)	$e^r_{si} = imp^s_{(ir)}$	$s = 1,\ldots,n_r+1$, $i = 1,..,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)^2$	Exports of the i'th commodity from the r'th region to the s'th region.
(4.15)	$e^r_i = \sum_s S^r_{Esi}\, e^r_{si}$	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)$	Exports of the i'th commodity from the r'th region.
(4.16)	$p^r_G = \sum_i S^{Dr}_{Gi}\, p^{Dr}_i + \sum_i \sum_s S^{Ir}_{G(is)}\, p^{Ir}_{(is)} + \sum_i (S^{Dr}_{Gi} + \sum_s S^{Ir}_{G(is)}) t^r_{Gi}$	$r = 1,\ldots,n_r$.	n_r	Price index for government usage of producer commodities.
(4.17)	$gov^{Ir}_{(is)} = g^r$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Government usage in the r'th region of the variety of the i'th commodity imported from the s'th region.
(4.18)	$dstoc^{Ir}_{(is)} = dstoc^{Tr}_R$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Change in stocks in the r'th region of the variety of the i'th commodity imported from the s'th region.
(4.19)	$imp^{n_r+1}_i = \varepsilon_i\,(p^{n_r+1}_{Wi} - p^{n_r+1}_{Ei})$	$i = 1,\ldots,n_s$.	n_s	Demand by the rest of the world for the composite imported variety of the i'th commodity.
(4.20)	$p^{n_r+1}_{Wi} = \sum_s S^{n_r+1}_{W(is)}\, p^{n_r+1}_{W(is)}$	$i = 1,\ldots,n_s$.	n_s	Price index for the composite variety of the i'th commodity imported into the rest of the world.
(4.21)	$imp^{n_r+1}_{(is)} = imp^{n_r+1}_i - \beta^{ROW}_i\,(p^{n_r+1}_{Wis} - p^{n_r+1}_{Wi})$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$.	$n_s(n_r+1)$	Demand by the rest of the world for the variety of the i'th commodity imported from the s'th region.
(4.22)	$p^r_{Wis} = S^r_{WB(is)}\,(p^s_{Ei} - er^s) + S^r_{WM(is)}\, p_T$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r+1$.	$n_s(n_r+1)^2$	Foreign currency LDF price in the r'th region of the variety of the i'th commodity imported from the s'th region.
(4.23)	$p^{Ir}_{(is)} = d^r_{(is)} + er^r + p^r_{Wis}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Basic price in the r'th region of the variety of the i'th commodity imported from the s'th region.
(4.24)	$pt^{Ir}_{(is)j} = t^{Ir}_{ij} + p^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Price paid by the j'th industry in the r'th region for the variety of the i'th commodity imported from the s'th region.

(Continued)

TABLE 4 : THE LINEARISED EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(4.25)	$p^{Dr}_{Ci} = t^{Dr}_{Ci} + p^{Dr}_{i}$	$i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r$	Price paid by households in the r'th region for the domestically produced variety of the i'th producer commodity.
(4.26)	$p^{Ir}_{C(is)} = t^{Ir}_{Ci} + p^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Price paid by households in the r'th region for the variety of the i'th producer commodity imported from the s'th region.
(4.27)	$p_T = \sum_r S_{Mr} (p^r_{E,n_s} - er^r)$		1	Price of international freight.
(4.28)	$imp^r_{(is)} = \sum_j S^{Ir}_{X(is)j} x^{Ir}_{(is)j} + S^{Ir}_{INV(is)} inv^{Ir}_{(is)} + S^{Ir}_{PC(is)} pc^{Ir}_{is} + S^{Ir}_{GOV(is)} gov_{(is)} + S^{Ir}_{DSTOC(is)} dstoc^{Ir}_{(is)}$	$s = 1,\ldots,n_r+1$, $i = 1,\ldots,n_s$, $r = 1,\ldots,n_r$.	$n_s n_r (n_r+1)$	Imports of the i'th commodity from the s'th region into the r'th region.
(4.29)	$p^r_I = \sum_i S^{Dr}_{Ki} p^{Dr}_i + \sum_i \sum_s S^{Ir}_{K(is)} p^{Ir}_{(is)} + \sum_i (S^{Dr}_{Ki} + \sum_s S^{Ir}_{K(is)}) t^r_{Ii}$	$r = 1,\ldots,n_r$.	n_r	Purchase price of capital in the r'th region.
(4.30)	$c^{Tr} = \sum_i S^{Dr}_{Ci} (p^{Dr}_{Ci} + pc^{Dr}_{Ci}) + \sum_i \sum_s S^{Ir}_{C(is)} (p^{Ir}_{C(is)} + pc^{Ir}_{(is)})$	$r = 1,\ldots,n_r$.	n_r	Aggregate consumption expenditure in the r'th region.
(4.31)	$cpi^r = \sum_i S^{Dr}_{Ci} p^{Dr}_{Ci} + \sum_i \sum_s S^{Ir}_{C(is)} p^{Ir}_{C(is)}$	$r = 1,\ldots,n_r$.	n_r	Consumer price index in the r'th region.
(4.32)	$dstoc^{Tr} = dstoc^{Tr}_R + dstocpi$	$r = 1,\ldots,n_r$.	n_r	Aggregate change in stocks in the r'th region at current prices.
(4.33)	$dstocpi^r = \sum_i S^D_{Si} p^{Dr}_i + \sum_i \sum_s S^I_{S(is)} p^{Ir}_{(is)} + \sum_i (S^D_{Si} + \sum_s S^I_{S(is)}) t^r_{Si}$	$r = 1,\ldots,n_r$.	n_r	Price index for change in stocks in the r'th region.
(4.34)	$imp^{Tr} = \sum_i \sum_s S^r_{IMP(is)} (p^r_{W(is)} + imp^r_{(is)})$	$r = 1,\ldots,n_r$.	n_r	Aggregate imports into the r'th region, at foreign currency LDF prices.

(Continued)

TABLE 4 : THE LINEARISED EQUATIONS OF THE INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(4.35)	$imppi^r = \sum_i \sum_s S^r_{IMP(is)} \, p^r_{W(is)}$	$r = 1,\ldots,n_r$.	n_r	Price index for imports into the r'th region.
(4.36)	$r^r_G = \sum_i \sum_s S^r_{RD(is)} \, (R^r_{RD(is)} \, d^r_{(is)} + er^r + p^r_{W(is)} + imp^r_{(is)})$	$r = 1,\ldots,n_r$.	n_r	Government revenue in the r'th region.
	$+ \sum_j \sum_i S^{Dr}_{RXij} \, (R^{Dr}_{RXij} + t^{Dr}_{ij} + p^{Dr}_i + x^{Dr}_{ij})$			
	$+ \sum_j \sum_i \sum_s S^{Ir}_{RX(is)j} \, (R^{Ir}_{RXij} \, t^{Ir}_{ij} + p^{Ir}_{(is)} + x^{Ir}_{(is)j})$			
	$+ \sum_i S^{Dr}_{RCi} \, (R^{Dr}_{RCi} \, t^{Dr}_{Ci} + p^{Dr}_{Ci} + pc^{Dr}_{Ci})$			
	$+ \sum_i \sum_s S^{Ir}_{RC(is)} \, (R^{Ir}_{RC(is)} \, t^{Ir}_{Ci} + p^{Ir}_{C(is)} + pc^{Ir}_{C(is)}$			
	$+ \sum_i S^r_{REi} \, (R^r_{REi} \, t^r_{Ei} + p^{Dr}_i + e^r_i)$			
	$+ \sum_i S^{Dr}_{RGi} \, (R^{Dr}_{RGi} \, t^r_{Gi} + p^{Dr}_i + gov^{Dr}_i)$			
	$+ \sum_i \sum_s S^{Ir}_{RG(is)} \, (R^{Ir}_{RG(is)} \, t^r_{Gi} + p^{Ir}_{(is)} + gov^{Ir}_{(is)})$			
	$+ \sum_i S^{Dr}_{RSi} \, (R^{Dr}_{RSi} \, t^r_{Si} + p^{Dr}_i + dstoc^{Dr}_i)$			
	$+ \sum_i \sum_s S^{Ir}_{RS(is)} \, (R^{Ir}_{RS(is)} \, t^r_{Si} + p^{Ir}_{(is)} + dstoc^{Ir}_{(is)})$			
	$- \sum_j S^r_{RBj} \, (R^r_{RBj} \, S^r_{Qj} + p^{Dr}_j + q^r_j)$			
	$+ S^r_{RY} \, S^r_{YM} \, (t^r_Y + y^r)$			

a n_s denotes the number of sectors in each region in the model. n_c denotes the number of consumer commodities. n_r denotes the number of regions.

TABLE 5: PARAMETERS UNIQUE TO THE LINEARISED INTER-REGIONAL LINKING EXTENSION TO WALRAS

Equation	Parameter	Description
(4.2)	$S^{Ir}_{(is)j}$	Share of the variety imported from the s'th region in intermediate usage expenditure on the composite imported variety of the i'th commodity by the j'th industry in the r'th region.
(4.5)	S^{r}_{Cig}	Share of the i'th producer commodity in consumption expenditure on the g'th consumer commodity in the r'th region.
(4.7)	S^{Dr}_{Cig}	Share of the domestically produced variety in consumption expenditure on the i'th producer commodity by way of the g'th consumer commodity in the r'th region.
(4.7)	S^{Ir}_{Cig}	Share of the composite imported variety in consumption expenditure on the i'th producer commodity by way of the g'th consumer commodity in the r'th region.
(4.10)	$S^{Ir}_{C(is)g}$	Share of the variety imported from the s'th region in consumption expenditure on the composite imported variety of the i'th producer commodity by way of the g'th consumer commodity in the r'th region.
(4.12)	S^{Dr}_{ig}	Share of consumption by way of the g'th consumer commodity in consumption of the domestically produced variety of the i'th producer commodity in the r'th region.
(4.13)	$S^{Ir}_{(is)g}$	Share of consumption by way of the g'th consumer commodity in consumption of the variety of the i'th producer commodity imported from the s'th region in the r'th region.
(4.15)	S^{r}_{Esi}	Share of the s'th region in exports of the i'th commodity from the r'th region.
(4.16)	$S^{Ir}_{G(is)}$	Share of the variety of the i'th commodity imported from the s'th region in government expenditure on producer commodities in the r'th region.
(4.20)	$S^{n_r+1}_{W(is)}$	Share of the s'th region in the LDF value of imports of the i'th commodity into the rest of the world.
(4.22)	$S^{r}_{WB(is)}$	Share of the FOB value in the LDF value of imports of the i'th commodity into the r'th region.
(4.22)	$S^{r}_{WM(is)}$	Share of international freight in the LDF value of imports of the i'th commodity into the r'th region.
(4.27)	S_{Mr}	Share of supply by the r'th region in the value of international freight.
(4.28)	$S^{Ir}_{X(is)j}$	Share of intermediate usage by the j'th industry in imports of the i'th commodity from the s'th region into the r'th region.
(4.28)	$S^{Ir}_{INV(is)}$	Share of investment usage in imports of the i'th commodity from the s'th region into the r'th region.
(4.28)	$S^{Ir}_{PC(is)}$	Share of consumption in imports of the i'th commodity from the s'th region into the r'th region.
(4.28)	$S^{Ir}_{GOV(is)}$	Share of government usage in imports of the i'th commodity from the s'th region into the r'th region.
(4.28)	$S^{Ir}_{DSTOC(is)}$	Share of change in stocks in imports of the i'th commodity from the s'th region into the r'th region.
(4.29)	$S^{Ir}_{K(is)}$	Share of the variety of the i'th commodity imported from the s'th region in the purchase price of capital in the r'th region.
(4.30), (4.31)	$S^{Ir}_{C(is)}$	Share of the variety of the i'th commodity imported from the s'th region in consumption expenditure in the r'th region.
(4.33)	$S^{Ir}_{S(is)}$	Share of the variety of the i'th commodity imported from the s'th region in inventory investment expenditure in the r'th region.
(4.34), (4.35)	$S^{r}_{IMP(is)}$	Share of imports of the i'th commodity from the s'th region in the LDF value of imports into the r'th region.
(4.36)	$S^{r}_{RD(is)}$	Share of duty on imports of the i'th commodity from the s'th region in government revenue in the r'th region.
(4.36)	$R^{r}_{RD(is)}$	Ratio of the power of the tariff to the tariff rate on imports of the i'th commodity from the s'th region into the r'th region.
(4.36)	$S^{Ir}_{RX(is)j}$	Share of tax on intermediate usage by the j'th industry of the variety of the i'th commodity imported from the s'th region in government revenue in the r'th region.

(Continued)

TABLE 5: PARAMETERS UNIQUE TO THE LINEARISED INTER-REGIONAL LINKING EXTENSION TO WALRAS (Continued)

Equation	Parameter	Description
(4.36)	$S^{Ir}_{RC(is)}$	Share of tax on consumption of the variety of the i'th commodity imported from the s'th region in government revenue in the r'th region.
(4.36)	$S^{Ir}_{RG(is)}$	Share of tax on government usage of the variety of the i'th commodity imported from the s'th region in government revenue in the r'th region.
(4.36)	$S^{Dr}_{RS(is)}$	Share of tax on change in stocks of the variety of the i'th commodity imported from the s'th region in government revenue in the r'th region.

SALTER WORKING PAPERS[a]

No.

1 McDougall, R. *Linking CGE Country Models: An Example Using the OECD's WALRAS Model*

2 McDougall, R. *Linking Regional Models in WALRAS*

3 McDougall, R. and Sugden, C. *Implementation of the WALRAS Model of the Australian Economy*

4 Zeitsch, J., McDougall, R., Jomini, P., Welsh, A., Hambley, J., Brown, S. and Kelly, J. *SALTER: A General Equilibrium of the World Economy*

5 Brown, S. *NERAM: A Nominal and Effective Rates of Assistance Model for the SALTER World Trade Model*

[a] SALTER working papers document work in progress on the development of the SALTER model of the world economy. They are made available to allow scrutiny of the work undertaken but should not be quoted without the permission of the author(s).