Implementation of the WALRAS Model of the Australian Economy

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1 Introduction

This memorandum describes the implementation by the Industries Assistance Commission (IAC) of the WALRAS model of the Australian economy.

WALRAS is an international general equilibrium model, developed by the Secretariat of the Organisation for Economic Co-Operation and Development (OECD). The model was designed to provide estimates of the effects of agricultural policies in the major agricultural trading regions of the OECD. The model contains six regional models, representing the economies of Australia, Canada, the European Economic Community, Japan, New Zealand and the United States of America. This memorandum describes the implementation by the IAC of one of the regional models.

WALRAS was designed as a long run, comparative static, non-linear model. In implementing the model the OECD Secretariat adopted a non-linear solution procedure. In implementing the Australian regional model the IAC has linearized it, and has employed the GEMPACK software system for linearized computable general equilibrium models (Codsi and Pearson, 1988).

No links exist at the time of writing between the regional models in WALRAS. The OECD Secretariat proposes to link the regional models, by adding equations relating each region's exports and export prices to the other regions' imports and import prices.

The existing state of the WALRAS model is described, and proposals for its future development are outlined, in a note by the OECD Secretariat.[1]

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1 OECD (1988a).

2. The Structure of the Model

2.1 Introduction

This section describes the structure of the IAC version of the WALRAS model of the Australian economy. The equations of the model are listed in Table 1, the variables in Table 2, and the parameters in Table 3.

This section describes the model in its original, non-linear form. The linearized equation system is presented in Section 3.

The text of this section briefly summarizes the equation system, which is intended to be consistent with the system implemented by the OECD (with one exception, noted in Section 2.10 below). It is not intended as a self-contained exposition of the equation system. The reader is referred to OECD (1988a) for a derivation of the demand equations from the underlying utility and production functions.

The model distinguishes three primary factors, thirteen single-product industries, and thirteen commodities. Each commodity is differentiated by source into a domestically produced and an imported variety. For the consumer demand system a different, ten commodity classification is adopted, with each commodity again being differentiated into domestically produced and imported varieties. The commodities defined by the ten commodity classification are called consumer commodities; where necessary for clarity, the commodities defined by the thirteen commodity classification are called producer commodities.

The model distinguishes five categories of final demand; investment, consumption, exports, government demand and change in stocks. However, government behaves in some respects more like a producer than like a source of final demand, since it demands not only commodities but also primary factors.

Since the equations presented in this section are expressed, not in percentage changes in variables, but in levels of variables, the units of each variable must be specified. The convention adopted here is to define units of each commodity such that the initial basic price of each

commodity is equal to one dollar. Units of primary factors are likewise defined such that the initial price of each factor to each industry is equal to one dollar.

The utility and production functions in WALRAS are nested. The arguments of the higher level functions in the nested system are composite commodities or composite factors, which are defined in the lower levels of the system by sub-production or sub-utility functions. Units of each composite commodity are defined such that the initial price of each composite commodity to its user is equal to one dollar.

The notation employed in this section is generally consistent with that in OECD (1988a). The main changes arise from the adoption in this memorandum of a convention that each symbol denoting the level of a variable should be in upper case; while the percentage change in the variable is denoted by the corresponding lower case symbol.

2.2 Demand for Inputs into Current Production

WALRAS distinguishes 13 single-product industries. In each industry, the production function is Leontieff in the production commodities and real value added. Usage of each commodity is defined as a constant-elasticity-of-substitution (CES) function of usage of the domestically produced and imported varieties of the commodity. Real value added is a CES function of usage of a labour-capital bundle and land. Usage of the labour-capital bundle is a CES function of usage of labour and capital.

Equations (2.1) to (2.6) of Table 1 are the demand equations for each industry's usage of each commodity (distinguished by source) and each primary factor. Demand for each commodity or factor is expressed as the product of an input-output coefficient and the level of industry output. These equations are definitional in nature; they implicitly define the input-output coefficients.

The behavioural content of the industry demand systems is in equations (2.7) to (2.10), (2.12) and (2.13). These equations express the input-output coefficients as functions of input prices. Because the production functions are linearly homogenous, the input-output coefficients do not depend upon output levels.

Finally, equation (2.11) defines the price of each industry's labour-capital bundle as a function of the usage rate and the rental price of capital.

2.3 Investment Demand

The WALRAS model recognizes just one capital good. Formation of this good is Leontieff in each production commodity (differentiated by source). Demands for each commodity (differentiated by source) for investment purposes are expressed in equations (2.14) and (2.15) as functions of aggregate gross real investment. These equations correspond to equation (1.47) of OECD (1988a). The variable INV_R^T, representing aggregate gross real investment, which does not appear explicitly in OECD (1988a), has been introduced into equations (2.14) and (2.15) for convenience. The proportionality constants r_i^D and r_i^I, listed by OECD (1988a) as variables, are treated here as parameters.

2.4 Consumer Demand

The consumer demand system is an extended linear expenditure system (ELES), in which the demands for ten consumer commodities and the supply of private savings are determined simultaneously. Consumption of each consumer commodity is defined by a CES function of consumption of the domestically produced and imported varieties of the commodity. Consumption of each consumer commodity from an individual source is Leontieff in the thirteen commodities from that source.

Equations (2.16) and (2.17) express demand for each consumer commodity, and the supply of private savings, as functions of aggregate disposable income and prices. Equation (2.18) expresses the price of each consumer commodity as a function of the prices of the domestically produced and

imported varieties of the commodity. Equations (2.19) and (2.20) express demand for the domestically produced and imported varieties of each consumer commodity as a function of demand for the composite consumer commodity and the prices of the two varieties. Finally, equations (2.21) and (2.22) express consumption demand for each producer commodity from each source as a function of demands for the consumer commodities from that source.

OECD (1988a) presents a similar set of equations, except that it does not present an equation corresponding to (2.18).

2.5 Export Demand

Equation (2.23) expresses the demand for Australian exports of each commodity as a function of the price of Australian exports, relative to the world price.

2.6 Government Demand

In WALRAS, the government demands not only producer commodities, but also labour and capital. The quantities demanded are such as to maximize an objective function which is CES in total usage of goods and services, usage of labour, and use of capital, subject to the constraint imposed by a fixed level of spending. Total usage of goods and services is Leontieff in the production commodities (differentiated by source).

Equations (2.24) to (2.26) express government demands for producer commodities (in aggregate), labour and capital as functions of total government spending, and the wage rate, the rental price of capital, and a price index for government usage of producer commodities. Equation (2.27) expresses the price index for government usage of producer commodities as a function of the price of each commodity (differentiated by source). Equations (2.28) and (2.29) express government demand for each commodity (differentiated by source) as a function of total government demand for producer commodities.

2.7 Change in Stocks

Equations (2.30) and (2.31) provide for flexible treatment of inventory investment. If the indexation parameter h_s is set equal to zero, then the composition of change in stocks is constant. This is the treatment suggested in OECD (1988a).

If the indexation parameter is set equal to one, then the share of change in stocks in total output (for domestically produced commodities) or imports (for imported commodities) is held constant.

2.8 The Price System

Equation (2.32) relates output prices to input prices, by imposing the condition that the sum of sales revenue and production subsidies in each industry is equal to total cost.

Equation (2.33) defines for convenience a vector variable which does not appear explicitly in OECD (1988a), namely the basic, or landed duty-paid, price of each imported commodity.

Equations (2.34) to (2.38) define purchasers' prices of intermediate inputs and consumer commodities, and the FOB price of exports, as functions of basic prices and commodity tax rates.

2.9 The Market-Clearing Equations

Equation (2.39) expresses the condition that supply of each domestically produced commodity is equal to demand. Equation (2.40), which is similar in form, is an identity, defining total imports of each commodity.

Equations (2.41) to (2.43) express the condition that demand for each primary factor is equal to endowment of the factor.

2.10 Miscellaneous Equations and Macro Identities

Equations (2.44) and (2.46) to (2.54) are identities defining components of aggregate demand (nominal and real), and associated price indexes. Similarly equations (2.56) to (2.58) define aggregate imports and the import price index (using foreign currency prices). Equation (2.45) defines the purchase price of capital. The constant RR, which is equal to the initial gross rate of return on capital, is required to convert the price from a price per unit of investment to a price per unit of capital.

Equation (2.58) is an identity defining the balance of trade at current domestic prices. Equation (2.59) defines a real balance of trade, using the import price index (converted to domestic currency) as a deflator. Equation (2.60) calculates the balance of trade as a percentage of NDP.

Equation (2.61) expresses the value of depreciation of the capital stock (at current prices) as the product of the depreciation rate and the value of the capital stock. This is the only deliberate inconsistency between the equation system described here and the system implemented by the OECD. In the system implemented by the OECD, depreciation is proportional, not to the rental price, but to the rate of return on capital.

Equations (2.62) to (2.67) define price indexes for and real levels of government spending and government outlays. The consumer price index is used to deflate the transfer component of government outlays. These equations also allow government spending and transfer payments to be held in fixed proportion to private spending and factor income respectively, if the user wishes. Equations (2.68) and (2.69) define nominal and real government revenue. The price index for government outlays is used to deflate government revenue. Equations (2.70) and (2.71) define nominal and real budget deficits (using the price index for government outlays as the deflator). Equation (2.72) calculates the budget deficit as a percentage of NDP.

Equations (2.73) to (2.76) define aggregate private income and aggregate private disposable income, net of depreciation. Equations (2.78) and (2.79) calculate net domestic product (NDP) from the expenditure side and the disposition side. If the data base is balanced, so that total sales are equal to total costs for each industry, then these two measures of net domestic product are equivalent. Equations (2.80) and (2.81) calculate a price index for NDP, and real NDP, both from the expenditure side.

Equation (2.77) defines a price index for gross domestic product at factor cost. This price index has been used as a deflator in results reported in OECD (1988b).

In equations (2.68) and (2.75), the intercept of the income tax function can (according to the setting of the indexation parameters h_{T1} and h_{T2}) be indexed to the CPI or to aggregate private income.

3. The Linearised WALRAS Model

As mentioned the version of WALRAS implemented by the IAC has been linearised to simplify the solution procedure. Table 4 presents the equation system of Table 1 in percentage change form. Table 5 presents the share coefficients and parameters unique to Table 4. The methodology for presenting some of the equations in a simplified form, for example CES functions, is adopted from Dixon et al. (1982).

The percentage change of a variable is denoted by the corresponding lower case symbol. That is;

$$y = 100 \left(\frac{dY}{Y}\right)$$

The only other difference between the non-linear and linear versions is that in the linearised version all tax rates appear as the power of the tax rather than the ad valorem rate.

The only variables in the linearised equation system which are not in percentage change form are those representing the balance of trade and the government budget deficit. Because these variables can change sign,

they are represented in the linearised system by changes rather than percentage changes in the original value of the variable. Thus delb and deld represent changes in the balance of trade and the budget deficit, measured in millions of dollars. The variables $delb_R$ and $deld_R$ represent changes in the real balance of trade and real budget deficit, measured in millions of dollars at initial prices. The variables delrbt and delrbd represent changes in the percentage ratio of the balance of trade and budget deficit to NDP.

4. Model Closure

In the model as described in Sections 2 and 3, the number of variables (counting each component of each subscripted variable separately) is equal to $8n_s^2 + 32n_s + 6n_c + 49$, where n_s denotes the number of industries or producer commodities in the model, and n_c denotes the number of consumer commodities. The number of equations is equal to $6n_s^2 + 23n_s + 6n_c + 40$.

In any simulation, the set of variables must be partitioned into an exogenous and an endogenous set, in such a way that the number of endogenous variables is equal to the number of equations. The number of exogenous variables must therefore be equal to $2n_s^2 + 9n_s + 9$.

In the sectoral classification adopted by the OECD, n_s is equal to thirteen, and n_c to 10. So the number of variables is 1877, and the number of equations is 1413. In any simulation there must be 464 exogenous variables and 1413 endogenous variables.

In the solution procedure implemented by the IAC, the partitioning of the variable list into exogenous and endogenous sets is determined by the user. This partitioning is called the model closure.

A list of exogenous variables for a typical closure of WALRAS is shown in Table 6. The exogenous variable list includes all direct and indirect tax rates, aggregate government spending and transfer payments (unadjusted for changes in the price level), aggregate employment of each factor of production, world prices, and the nominal exchange rate. It also includes aggregate real investment. It does not include

the balance of trade or the budget deficit. The inclusion of real investment in the exogenous variable list, contrary to OECD (1988a) para 41, is explained in the appendix.

Besides a typical closure, OECD (1988a) describes certain alternative closures. One alternative is to fix the budget deficit, while endogenising the marginal rate of income tax. Another alternative is to fix the balance of trade, while endogenizing the nominal exchange rate. This suggestion may appear strange to users of standard ORANI closures, in which the nominal exchange rate serves as numeraire. In those closures real variables are influenced by changes in relative prices, but not by changes in the price level. But in the closures suggested in OECD (1988a), changes in the price level do affect real variables, since some nominal variables are exogenous (aggregate government spending, transfer payments, and in some closures the budget deficit or the balance of trade (at domestic currency prices)). So change in the nominal exchange rate leads not only to change in the price level but also to changes in real variables, and therefore to change in the balance of trade at foreign currency prices.

5. The Condensed Version of the Model

The model as described in previous sections contains a large number of variables, many of which are of slight interest in most simulations. In the process of implementation the model has been condensed, by eliminating many variables which are likely to be endogenous, and of minor interest, in most applications of the model. Condensation eliminates equal numbers of endogenous variables and equations from the model.

Table 7 shows the variables retained in the condensed version of the model. The number of retained variables is $2n_s^2 + 19n_s + 2n_c + 49$, and the number of eliminated variables is $6n_s^2 + 13n_s + 4n_c$. Condensation reduces the number of equations and endogenous variables to $10n_s + 2n_c + 40$. With $n_s = 13$ and $n_c = 10$, condensation reduces the total number of variables to 654 and the number of endogenous variables to 190. The number of exogenous variables is unchanged.

6. The Data Base

The data base for the WALRAS model of the Australian economy was implemented with the GEMPACK program MODHAR (documented in Pearson (1986)). The data based consists of two header array files, one containing the input-output data required to calculate the share parameters, and the other containing the behavioural parameters. The contents of these files are listed in Tables 8 and 9.

Identifier	Equation	Subscript range[a]	Number[a]	Description
(2.1)	$X^D_{ij} = A^D_{ij} Q_j$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Demand by the j'th industry for the i'th domestically produced commodity.
(2.2)	$X^I_{ij} = A^I_{ij} Q_j$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Demand by the j'th industry for the i'th imported commodity.
(2.3)	$H_j = A_{Hj} Q_j$	$j = 1,\ldots,n_s$.	n_s	Demand by the j'th industry for its labour-capital bundle.
(2.4)	$M_j = A_{Mj} Q_j$	$j = 1,\ldots,n_s$.	n_s	Demand by the j'th industry for land.
(2.5)	$L_j = A_{Lj} Q_j$	$j = 1,\ldots,n_s$.	n_s	Demand by the j'th industry for labour.
(2.6)	$K_j = A_{Kj} Q_j$	$j = 1,\ldots,n_s$.	n_s	Demand by the j'th industry for capital.
(2.7)	$A^D_{ij} = (Y^D_{ij})^{\eta_i} (PT^D_{ij})^{-\eta_i} \left[(Y^D_{ij})^{\eta_i} (PT^D_{ij})^{1-\eta_i} + (Y^I_{ij})^{\eta_i} (PT^I_{ij})^{1-\eta_i}\right]^{\eta_i/(1-\eta_i)} A_{ij}$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Input-output coefficient for usage by the j'th industry of the i'th domestically produced commodity.
(2.8)	$A^I_{ij} = (Y^I_{ij})^{\eta_i} (PT^I_{ij})^{-\eta_i} \left[(Y^D_{ij})^{\eta_i} (PT^D_{ij})^{1-\eta_i} + (Y^I_{ij})^{\eta_i} (PT^I_{ij})^{1-\eta_i}\right]^{\eta_i/(1-\eta_i)} A_{ij}$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Input-output coefficient for usage by the j'th industry of the i'th imported commodity.
(2.9)	$A_{Hj} = \alpha^{\rho_j}_{Hj} RH_j^{-\rho_j} (\alpha^{\rho_j}_{Hj} RH_j^{1-\rho_j} + \alpha^{\rho_j}_{Mj} RM^{1-\rho_j})^{\rho_j/(1-\rho_j)} A_{Vj}$	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of its labour-capital bundle.
(2.10)	$A_{Mj} = \alpha^{\rho_j}_{Mj} RM^{-\rho_j} (\alpha^{\rho_j}_{Hj} RH_j^{1-\rho_j} + \alpha^{\rho_j}_{Mj} RM^{1-\rho_j})^{\rho_j/(1-\rho_j)} A_{Vj}$	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of land.
(2.11)	$RH_j = (\alpha^{\sigma_j}_{Lj} W^{1-\sigma_j} + \alpha^{\sigma_j}_{Kj} R^{1-\sigma_j})^{1/(1-\sigma_j)}$	$j = 1,\ldots,n_s$.	n_s	Price index for the labour-capital bundle used by the j'th industry.
(2.12)	$A_{Lj} = \alpha^{\sigma_j}_{Lj} W^{-\sigma_j} (\alpha^{\sigma_j}_{Lj} W^{1-\sigma_j} + \alpha^{\sigma_j}_{Kj} R^{1-\sigma_j})^{\sigma_j/(1-\sigma_j)} A_{Hj}$	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of labour.
(2.13)	$A_{Kj} = \alpha^{\sigma_j}_{Kj} R^{-\sigma_j} (\alpha^{\sigma_j}_{Lj} W^{1-\sigma_j} + \alpha^{\sigma_j}_{Kj} R^{1-\sigma_j})^{\sigma_j/(1-\sigma_j)} A_{Hj}$	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of capital.

(Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(2.14)	$INV_i^D = r_i^D \, INV_R^T$	$i = 1,\ldots,n_s$.	n_s	Investment usage of the i'th domestically produced commodity.
(2.15)	$INV_i^I = r_i^I \, INV_R^T$	$i = 1,\ldots,n_s$.	n_s	Investment usage of the i'th imported commodity.
(2.16)	$C_g = F_g + \theta_g (YD - \sum_h P_{Ch} F_h)/P_{Cg}$	$g = 1,\ldots,n_c$.	n_c	Demand for the g'th consumer commodity.
(2.17)	$SV = \theta_{11} (YD - \sum_g P_{Cg} F_g)$		1	Supply of private savings.
(2.18)	$P_{Cg} = [(\omega_g^D)^{\beta_g} (P_{Cg}^D)^{1-\beta_g} + (\omega_g^I)^{\beta_g} (P_{Cg}^I)^{1-\beta_g}]^{1/(1-\beta_g)}$	$g = 1,\ldots,n_c$.	n_c	Price index for the g'th consumer commodity.
(2.19)	$C_g^D = (\omega_g^D)^{\beta_g} (P_{Cg}^D)^{-\beta_g} [(\omega_g^D)^{\beta_g} (P_{Cg}^D)^{1-\beta_g} + (\omega_g^I)^{\beta_g} (P_{Cg}^I)^{1-\beta_g}]^{\beta_g/(1-\beta_g)} C_g$	$g = 1,\ldots,n_c$.	n_c	Demand for the g'th domestically produced consumer commodity.
(2.20)	$C_g^I = (\omega_g^I)^{\beta_g} (P_{Cg}^I)^{-\beta_g} [(\omega_g^D)^{\beta_g} (P_{Cg}^D)^{1-\beta_g} + (\omega_g^I)^{\beta_g} (P_{Cg}^I)^{1-\beta_g}]^{\beta_g/(1-\beta_g)} C_g$	$g = 1,\ldots,n_c$.	n_c	Demand for the g'th imported consumer commodity.
(2.21)	$PC_i^D = \sum_g \xi_{ig}^D C_g^D$	$i = 1,\ldots,n_s$.	n_s	Household consumption of the i'th domestically produced producer commodity.
(2.22)	$PC_i^I = \sum_g \xi_{ig}^I C_g^I$	$i = 1,\ldots,n_s$.	n_s	Household consumption of the i'th imported producer commodity.
(2.23)	$E_i = \delta_i \cdot (P_{Ei}/ER.\ P_{Wi})^{\epsilon_i}$	$i = 1,\ldots,n_s$.	n_s	Exports of the i'th commodity.
(2.24)	$G = \phi_G^\psi P_G^{-\psi} (\phi_G^\psi P_G^{1-\psi} + \phi_L^\psi W^{1-\psi} + \phi_K^\psi R^{1-\psi})^{-1} Z_G$		1	Total real government usage of producer commodities.
(2.25)	$L_G = \phi_L^\psi W^{-\psi} (\phi_G^\psi P_G^{1-\psi} + \phi_L^\psi W^{1-\psi} + \phi_K^\psi R^{1-\psi})^{-1} Z_G$		1	Government usage of labour.
(2.26)	$K_G = \phi_K^\psi R^{-\psi} (\phi_G^\psi P_G^{1-\psi} + \phi_L^\psi W^{1-\psi} + \phi_K^\psi R^{1-\psi})^{-1} Z_G$		1	Government usage of capital.
(2.27)	$P_G = \sum_i \chi_i^D (1 + t_{Gi}) P_i^D + \sum_i \chi_i^I (1 + t_{Gi}) P_i^I$		1	Price index for government usage of producer commodities.
(2.28)	$GOV_i^D = \chi_i^D G$	$i = 1,\ldots,n_s$.	n_s	Government usage of the i'th domestically produced commodity.

(Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(2.29)	$GOV_i^I = \chi_i^I G$	$i = 1,\ldots,n_s$.	n_s	Government usage of the i'th imported commodity.
(2.30)	$DSTOC_i^D = F_s A_{Si}^D Q_i^{n_s}$	$i = 1,\ldots,n_s$.	n_s	Change in stocks of the i'th domestically produced commodity.
(2.31)	$DSTOC_i^I = F_s A_{Si}^I IMP_i^{n_s}$	$i = 1,\ldots,n_s$.	n_s	Change in stocks of the i'th imported commodity.
(2.32)	$P_j^D Q_j(1 + S_{Qj}) = \sum_i PT_{ij}^D X_{ij}^D + \sum_j PT_{ij}^I X_{ij}^I + W.L_j + R.K_j + RM.M_j$	$j = 1,\ldots,n_s$.	n_s	Zero profits condition for the j'th industry.
(2.33)	$P_i^I = (1 + D_i)\ ER.P_{Wi}$	$i = 1,\ldots,n_s$.	n_s	Basic price of the i'th imported commodity.
(2.34)	$PT_{ij}^D = (1 + T_{ij}^D)\ P_i^D$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Price paid by the j'th industry for the i'th domestically produced commodity.
(2.35)	$PT_{ij}^I = (1 + T_{ij}^I)\ P_i^I$	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Price paid by the j'th industry for the i'th imported commodity.
(2.36)	$P_{Cg}^D = \sum_i \xi_{ig}^D (1 + T_{Ci}^D)\ P_i^D$	$g = 1,\ldots,n_c$.	n_c	Price paid by households for the i'th domestically produced consumer commodity.
(2.37)	$P_{Cg}^I = \sum_i \xi_{ig}^D (1 + T_{Ci}^I)\ P_i^I$	$g = 1,\ldots, n_c$	n_c	Price paid by households for the i'th imported consumer commodity.
(2.38)	$P_{Ei} = (1 + T_{Ei})\ P_i^D$	$i = 1,\ldots,n_s$	n_s	FOB price of exports of the i'th commodity.
(2.39)	$Q_i = \sum_i X_{ij}^D + INV_i^D + PC_i^D + E_i + GOV_i^D + DSTOC_i^D$	$i = 1,\ldots,n_s$	n_s	Market clearing condition for the i'th domestically produced commodity.
(2.40)	$IMP_i = \sum_j X_{ij}^I + INV_i^I + PC_i^I + GOV_i^I + DSTOC_i^I$	$i = 1,\ldots,n_s$.	n_s	Imports of the i'th commodity.
(2.41)	$L^T = \sum_j L_j + L_g$		1	Aggregate usage of labour.
(2.42)	$K^T = \sum_j K_j + K_G$		1	Aggregate usage of capital.
(2.43)	$M^T = \sum_j M_j$		1	Aggregate usage of land.

(Continued)

TABLE 1 : THE WALRAS EQUATIONS (Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(2.44)	$INV^T = P_I \, INV_R^T$		1	Aggregate investment at current prices.
(2.45)	$P_I = \{\sum_i \Gamma_i^D (1 + T_{Ii}) P_i^D + \sum_i \Gamma_i^I (1 + T_{Ii}) P_i^I\}/RR$		1	Purchase price of capital.
(2.46)	$C^T = \sum_g P_{Cg}^D C_g^D + \sum_g P_{Cg}^I C_g^I$		1	Aggregate consumption at current prices.
(2.47)	$CPI = \sum_g A_{Cg}^D P_{Cg}^D + \sum_g A_{Cg}^I P_{Cg}^I$		1	Consumer price index.
(2.48)	$C_R^T = C^T/CPI$		1	Real aggregate consumption.
(2.49)	$E^T = \sum_i P_{Ei} E_i$		1	Aggregate exports at current FOB prices.
(2.50)	$EPI = \sum_i A_{Ei} P_{Ei}$		1	Export price index (FOB prices).
(2.51)	$E_R^T = E^T/EPI$		1	Real aggregate exports.
(2.52)	$DSTOC^T = \sum_i (1 + T_{Si}) P_i^D DSTOC_i^D + \sum_i (1 + T_{Si}) P_i^I DSTOC_i^I$		1	Aggregate change in stocks at current prices.
(2.53)	$DSTOCPI = \sum_i \Omega_i^D (1 + T_{Si}) P_i^D + \sum_i \Omega_i^I (1 + T_{Si}) P_i^I$		1	Price index for change in stocks.
(2.54)	$DSTOC_R^T = DSTOC^T/DSTOCPI$		1	Real change in stocks.
(2.55)	$IMP^T = \sum_i P_{Wi} IMP_i$		1	Aggregate imports at current foreign currency prices.
(2.56)	$IMPPI = \sum_i A_{Ii} P_{Wi}$		1	Import price index (world prices).
(2.57)	$IMP_R^T = IMP^T/IMPPI$		1	Real aggregate imports.
(2.58)	$BT = E^T - ER.IMP^T$		1	Balance of trade, at current domestic prices.
(2.59)	$BT_R = BT/ER.IMPPI$		1	Real balance of trade.
(2.60)	$RBT = 100.BT/NDP_E$		1	Ratio of the balance of trade to NDP.
(2.61)	$DEP = \sum_i \Gamma_i^T$		1	Depreciation of the capital stock.

TABLE 1 : THE WALRAS EQUATIONS (Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(2.62)	$ZPI = A_{ZG} P_G + A_{ZL} W + A_{Zn} R$		1	Price index for government spending.
(2.63)	$Z_{GR} = Z_G/ZPI$		1	Real government spending.
(2.64)	$Z_G = F_Z.C^T$		1	Government spending relative to private consumption spending.
(2.65)	$T_{GR} = T_G/CPI$		1	Real transfer payments by government.
(2.66)	$T_G = F_{TG}.Y_{FN}$		1	Transfer payments by government, relative to factor income.
(2.67)	$OPI = A_{OZ}\ ZPI + A_{OT}\ CPI$		1	Price index for government outlays.
(2.68)	$R_G = \sum_i D_i\ ER.P_{Wi} IMP_i + \sum_i \sum_j T^D_{ij} P^D_i X^D_{ij} + \sum_i \sum_j T^I_{ij} P^I_i X^I_{ij} + \sum_i T_{Ii} P^D_i INV^D_i + \sum_i T_{Ii} P^I_i INV^I_i$ $+ \sum_i T^D_{Ci} P^D_i PC^D_i + \sum_i T^I_{Ci} P^I_i PC^I_i + \sum_i T_{Ei} P^D_i E_i$ $+ \sum_i T_{Gi} P^D_i GOV^D_i + \sum_i T_{Gi} P^I_i GOV^I_i$ $+ \sum_i T_{Si} P^D_i DSTOC^D_i + \sum_i T_{Si} P^I_I DSTOC^I_i$ $- \sum_j S_{Qj} P^D_j Q_j + (CPI)^{h_{T1}} y^{h_{T2}} \tau_y + T_y Y$		1	Government revenue.
(2.69)	$R_{GR} = R_G/OPI$		1	Real government revenue.
(2.70)	$BD = Z_G + T_G - R_G$		1	Budget deficit.
(2.71)	$BD_R = BD/OPI$		1	Real budget deficit.
(2.72)	$RBD = 100.BD/NDP_E$		1	Ratio of budget deficit to NDP.
(2.73)	$Y_{FN} = W.L^T + R.K^T + RM.M^T - DEPR$		1	Aggregate factor income.
(2.74)	$Y = Y_{FN} + T_G$		1	Aggregate private income.
(2.75)	$YD = (1 - T_y)\ Y - (CPI)^{h_{T1}} Y^{h_{T2}} \tau_y$		1	Aggregate household disposable income.

(Continued)

Identifier	Equation	Subscript range[a]	Number[a]	Description
(2.76)	$YD_R = YD/CPI$		1	Real aggregate disposable income.
(2.77)	$GDPPI_{FC} = A_{GL} W + A_{GK} R + A_{GM} RM$		1	Price index for GDP at factor cost.
(2.78)	$NDP_E = C^T + (INV^T - DEPR) + Z_G + (E^T - ER.IMP^T) + DSTOC^T$		1	Net domestic product from the expenditure side.
(2.79)	$NDP_D = C^T + SV + (R_G - T_G)$		1	Net domestic product from the disposition side.
(2.80)	$NDPPI_E = A_{NC} CPI + A_{NI} PI + A_{NZ} ZPI + A_{NE} EPI + A_{NM} IMPPI. ER + A_{NS} DSTOCPI$		1	Price index for NDP (calculated from the expenditure side).
(2.81)	$NDP_{ER} = NDP_E/NDPPI_E$		1	Real NDP (calculated from the expenditure side).

a n_s denotes the number of sectors in the model. n_c denotes the number of consumer commodities.

Variable	Subscript range[a]	Number[a]	Description
A^D_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Input-output coefficient for usage by the j'th industry of the i'th domestically produced commodity.
A^I_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Input-output coefficient for usage by the j'th industry of the i'th imported commodity.
PT^D_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Price paid by the j'th industry for the i'th domestically produced commodity.
PT^I_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Price paid by the j'th industry for the i'th imported commodity.
T^D_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Commodity tax rate on usage by the j'th industry of the i'th domestically produced commodity.
T^I_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Commodity tax rate on usage by the j'th industry of the i'th imported commodity.
X^D_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Usage by the j'th industry of the i'th domestically produced commodity.
X^I_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Usage by the j'th industry of the i'th imported commodity.
PC^D_i	$i = 1,\ldots,n_s$.	n_s	Consumption of the i'th domestically produced commodity.
PC^I_i	$i = 1,\ldots,n_s$.	n_s	Consumption of the i'th imported commodity.
D_i	$i = 1,\ldots,n_s$.	n_s	Tariff rate on imports of the i'th commodity.
$DSTOC^D_i$	$i = 1,\ldots,n_s$.	n_s	Change in stocks of the i'th domestically produced commodity.
$DSTOC^I_i$	$i = 1,\ldots,n_s$.	n_s	Change in stocks of the i'th imported commodity.
E_i	$i = 1,\ldots,n_s$.	n_s	Exports of the i'th commodity.
GOV^D_i	$i = 1,\ldots,n_s$.	n_s	Government usage of the i'th domestically produced commodity.
GOV^I_i	$i = 1,\ldots,n_s$.	n_s	Government usage of the i'th imported commodity.
IMP_i	$i = 1,\ldots,n_s$.	n_s	Imports of the i'th commodity.
INV^D_i	$i = 1,\ldots,n_s$.	n_s	Investment usage of the i'th domestically produced commodity.
INV^I_i	$i = 1,\ldots,n_s$.	n_s	Investment usage of the i'th imported commodity.
P^D_i	$i = 1,\ldots,n_s$.	n_s	Basic price of the i'th domestically produced commodity.
P^I_i	$i = 1,\ldots,n_s$.	n_s	Basic price of the i'th imported commodity.
P_{Ei}	$i = 1,\ldots,n_s$.	n_s	Export price of the i'th commodity.
P_{Wi}	$i = 1,\ldots,n_s$.	n_s	World price of the i'th commodity.
T^D_{Ci}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on consumption of the i'th domestically produced commodity.
T^I_{Ci}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on consumption of the i'th imported commodity.
T_{Ei}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on exports of the i'th commodity.
T_{Gi}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on government usage of the i'th commodity.
T_{Ii}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on investment usage of the i'th commodity.
T_{Si}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on change in stocks of the i'th commodity.
A_{Hj}	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of its labour-capital bundle.
A_{Kj}	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of capital.
A_{Lj}	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of labour.
A_{Mj}	$j = 1,\ldots,n_s$.	n_s	Input-output coefficient for usage by the j'th industry of land.
H_j	$j = 1,\ldots,n_s$.	n_s	Usage by the j'th industry of its labour-capital bundle.
K_j	$j = 1,\ldots,n_s$.	n_s	Usage by the j'th industry of capital.
L_j	$j = 1,\ldots,n_s$.	n_s	Usage by the j'th industry of labour.

(Continued)

Variable	Subscript range[a]	Number[a]	Description
M_j	$j = 1,\ldots,n_s$.	n_s	Usage by the j'th industry of agricultural land.
Q_j	$j = 1,\ldots,n_s$.	n_s	Output of the j'th industry.
RH_j	$j = 1,\ldots,n_s$.	n_s	Shadow price of the labour-capital bundle used in the j'th industry.
S_{Qj}	$j = 1,\ldots,n_s$.	n_s	Production subsidy rate on output of the j'th industry.
C_g	$g = 1,\ldots,n_c$.	n_c	Consumption of the g'th consumer commodity.
C_g^D	$g = 1,\ldots,n_c$.	n_c	Consumption of the g'th domestically produced consumer commodity.
C_g^I	$g = 1,\ldots,n_c$.	n_c	Consumption of the g'th imported consumer commodity.
P_{Cg}	$g = 1,\ldots,n_c$.	n_c	Shadow price of the g'th consumer commodity.
P_{Cg}^D	$g = 1,\ldots,n_c$.	n_c	Price of the g'th domestically produced consumer commodity.
P_{Cg}^I	$g = 1,\ldots,n_c$.	n_c	Price of the g'th imported consumer commodity.
BD		1	Budget deficit.
BD_R		1	Real budget deficit.
BT		1	Balance of trade.
BT_R		1	Real balance of trade.
C^T		1	Aggregate consumption.
C_R^T		1	Real aggregate consumption.
CPI		1	Consumer price index.
DEPR		1	Depreciation of capital.
$DSTOC^T$		1	Aggregate change in stocks, at current prices.
$DSTOC_R^T$		1	Aggregate real change in stocks.
DSTOCPI		1	Price index for change in stocks.
E^T		1	Aggregate exports, at current prices (domestic currency).
E_R^T		1	Real aggregate exports.
EPI		1	Export price index (FOB prices).
ER		1	Exchange rate (units of domestic currency per unit of foreign currency).
F_s		1	Shift term for change in stocks.
F_Z		1	Ratio of aggregate government spending to private consumption spending.
F_{TG}		1	Ratio of transfer payments by government to net factor income.
G		1	Total real government usage of producer commodities.
$GDPPI_{FC}$		1	Price index for GDP at factor cost.
IMP^T		1	Aggregate imports at current foreign currency prices.
IMP_R^T		1	Aggregate real imports.

(Continued)

TABLE 2 : THE MODEL VARIABLES (continued)

Variable	Subscript range[a]	Number[a]	Description
IMPPI		1	Import price index (foreign currency units).
INV^T		1	Aggregate gross investment at current prices.
INV_R^T		1	Aggregate gross real investment.
K^T		1	Aggregate usage of capital.
K_G		1	Government usage of capital.
L^T		1	Aggregate usage of labour.
L_G		1	Government usage of labour.
M^T		1	Aggregate usage of land.
NDP_D		1	NDP, calculated from the disposition side, at current prices.
NDP_E		1	NDP, calculated from the expenditure side, at current prices.
NDP_{ER}		1	Real net domestic product, calculated from the expenditure side.
$NDPPI_E$		1	Price index for net domestic product, calculated from the expenditure side.
OPI		1	Price index for government outlays.
P_G		1	Price index for government usage of producer commodities.
P_I		1	Purchase price of capital.
R		1	Rental price of capital.
R_G		1	Government revenue.
R_{GR}		1	Real government revenue.
RBD		1	Ratio of the budget deficit to NDP.
RBT		1	Ratio of the balance of trade to NDP.
RM		1	Rental price of land.
SV		1	Private saving.
T_G		1	Transfer payments by government.
T_{GR}		1	Real transfer payments by government.
T_Y		1	Marginal income tax rate.
W		1	Wage rate.
Y		1	Aggregate income.
YD		1	Aggregate disposable income.
YD_R		1	Real aggregate disposable income.
Y_{FN}		1	Net factor income.
Z_G		1	Government spending.
Z_{GR}		1	Real government spending.
ZPI		1	Price index for government spending.

a n_s denotes the number of sectors in the model. n_c denotes the number of consumer commodities.

(2.7), (2.8)	γ^D_{ij}	Intensity of usage by the j'th industry of the i'th commodity with respect to the i'th domestically produced commodity.	Calibration. $\gamma^D_{ij} = (A^D_{ij}/A_{ij})^{1/\eta_i} PT^D_{ij}$
(2.7), (2.8)	γ^I_{ij}	Intensity of usage by the j'th industry of the i'th commodity with respect to the i'th imported commodity.	Calibration. $\gamma^I_{ij} = (A^I_{ij}/A_{ij})^{1/\eta_i} PT^I_{ij}$
(2.7), (2.8)	η_i	Elasticity of substitution in intermediate usage between the i'th domestically produced commodity and the i'th imported commodity.	Elasticities file.
(2.7), (2.8)	A_{ij}	Input-output coefficient for usage by the j'th industry of the i'th commodity.	Calibration. $A_{ij} = (PT^D_{ij} X^D_{ij} + PT^I_{ij} X^I_{ij})/P_j Q_j$
(2.9), (2.10)	α_{Hj}	Intensity of value added in the j'th industry with respect to its labour-capital bundle.	Calibration. $\alpha_{Hj} = (A_{Hj}/A_{Vj})^{1/\rho_j}$
(2.9), (2.10)	α_{Mj}	Intensity of value added in the j'th industry with respect to land.	Calibration. $\alpha_{Mj} = (A_{Mj}/A_{Vj})^{1/\rho_j}$
(2.9), (2.10)	ρ_j	Elasticity of substitution in the j'th industry between the labour-capital bundle and land.	Elasticities file.
(2.9), (2.10)	A_{Vj}	Input-output coefficient for value added in the j'th industry.	Calibration.
(2.11)-(2.13)	α_{Lj}	Intensity of the labour-capital bundle in the j'th industry with respect to labour.	Calibration. $\alpha_{Lj} = (A_{Lj}/A_{Hj})^{1/\sigma_j}$
(2.11)-(2.13)	α_{Kj}	Intensity of the labour-capital bundle in the j'th industry with respect to labour.	Calibration. $\alpha_{Kj} = (A_{Kj}/A_{Hj})^{1/\sigma_j}$
(2.11)-(2.13)	σ_j	Elasticity of substitution in the j'th industry between labour and capital.	Elasticities file.
(2.14), (2.45)	Γ^D_i	Input-output coefficient for usage of the i'th domestically produced commodity in capital creation.	Calibration.
(2.15), (2.45)	Γ^I_i	Input-output coefficient for usage of the i'th imported commodity in capital creation.	Calibration.
(2.16), (2.17)	F_g	Subsistance consumption of the g'th consumer commodity.	Calibration. $Fg = Cg + \theta_g YD/\omega$
(2.16)	θ_g	Marginal budget share of the g'th commodity.	Calibration. (Income elasticity) x (Average budget share)
	ω	Frisch parameter.	Elasticities file.
(2.17)	θ_{11}	Marginal propensity to save.	Elasticities file.

(Continued)

Equation	Parameter	Description	Source
(2.18)-(2.20)	ω_g^D	Intensity of consumption of the g'th consumer commodity with respect to the g'th domestically produced consumer commodity.	Calibration. $\omega_g^D = (C_g^D/C_g)^{1/\beta_g}$
(2.18)-(2.20)	ω_g^I	Intensity of consumption of the g'th consumer commodity with respect to the g'th imported consumer commodity.	Calibration. $\omega_g^I = (C_g^I/C_g)^{1/\beta_g}$
(2.18)-(2.20)	β_g	Elasticity of substitution between the g'th domestically produced and the g'th imported consumer commodities.	Elasticities file.
(2.21), (2.36)	ξ_{ig}^D	Input-output coefficient for the i'th domestically produced commodity and the g'th domestically produced consumer commodity.	Calibration.
(2.22), (2.37)	ξ_{ig}^I	Input-output coefficient for the i'th imported commodity and the g'th imported consumer commodity.	Calibration.
(2.23)	δ_i	Scale parameter for exports of the i'th commodity.	Calibration. $\delta_i = E_i (P_{Ei}/P_{wi})^{-\varepsilon_i}$
(2.23)	ε_i	Elasticity of export demand for the i'th commodity.	Elasticities file.
(2.24)-(2.26)	ϕ_G	Intensity of government spending with respect to producer commodities	Calibration. $\phi_G = (G/Z_G)^{1/\psi}$
(2.24)-(2.26)	ϕ_L	Intensity of government spending with respect to labour.	Calibration. $\phi_L = (L_G/Z_G)^{1/\psi}$
(2.24)-(2.26)	ϕ_K	Intensity of government spending with respect to capital.	Calibration. $\phi_K = (K_G/Z_G)^{1/\psi}$
(2.24)-(2.26)	ψ	Elasticity of substitution between producer commodities, labour and capital in government spending.	Elasticities file.
(2.27), (2.28)	x_i^D	Input-output coefficient for the i'th domestically produced commodity and total government spending on producer commodities.	Calibration.
(2.27), (2.29)	x_i^I	Input-output coefficient for the i'th imported commodity and total government spending on producer commodities.	Calibration.
(2.30)	A_{Si}^D	Proportionality constant for change in stocks of the i'th domestically produced commodity.	Calibration.
(2.30), (2.31)	h_s	Indexation parameter for change in stocks and aggregate output or imports.	Elasticities file.

(Continued)

Equation	Parameter	Description	Source
(2.31)	A^{I}_{3i}	Proportionality constant for change in stocks of the i'th imported commodity.	Calibration.
(2.45)	RR	Initial gross rate of return on capital.	Elasticities file.
(2.47)	A^{D}_{Cg}	Average budget share of the g'th domestically produced consumer commodity.	Calibration.
(2.47)	A^{I}_{Cg}	Average budget share of the g'th imported consumer commodity.	Calibration.
(2.48)	A_{Ei}	Share of the i'th commodity in exports.	Calibration.
(2.56)	A_{Ii}	Share of the i'th commodity in imports.	Calibration.
(2.61)	ζ	Rate of depreciation of capital.	Elasticities file.
(2.62)	A_{ZG}	Share of producer commodities in government spending.	Calibration.
(2.62)	A_{ZL}	Share of labour in government spending.	Calibration.
(2.62)	A_{ZK}	Share of capital services in government spending.	Calibration.
(2.67)	A_{OZ}	Share of spending in government outlays.	Calibration.
(2.67)	A_{OT}	Share of transfers in government outlays.	Calibration.
(2.68), (2.75)	τ_y	Intercept of the direct tax function.	Elasticities file.
(2.68), (2.75)	n_{T1}	Parameter indexing income tax to the CPI.	Elasticities file.
(2.68), (2.75)	n_{T2}	Parameter indexing income tax to aggregate private income.	Elasticities file.
(2.77)	A_{GL}	Share of labour in GDP at factor cost.	Calibration.
(2.77)	A_{GK}	Share of capital in GDP at factor cost.	Calibration.
(2.77)	A_{GM}	Share of land in GDP at factor cost.	Calibration.
(2.80)	A_{NC}	Share of consumption in NDP.	Calibration.
(2.80)	A_{NI}	Share of net investment in NDP.	Calibration.
(2.80)	A_{NZ}	Share of government spending in NDP.	Calibration.
(2.80)	A_{NE}	Share of export in NDP.	Calibration.
(2.80)	A_{NM}	Share of imports in NDP.	Calibration.
(2.80)	A_{NS}	Share of change in stocks in NDP.	Calibration.

Identifier	Equation			Subscript range		
(4.1)	x^D_{ij}	=	$a^D_{ij} + q_j$	$i = 1,\ldots, 13$, $j = 1,\ldots, 13$.	13^2	Demand by the j'th industry for the i'th domestically produced commodity.
(4.2)	x^I_{ij}	=	$a^I_{ij} + q_j$	$i = 1,\ldots, 13$, $j = 1,\ldots, 13$.	13^2	Demand by the j'th industry for the i'th imported commodity.
(4.3)	h_j	=	$a_{Hj} + q_j$	$j = 1,\ldots, 13$.	13	Demand by the j'th industry for its labour-capital bundle.
(4.4)	m_j	=	$a_{Mj} + q_j$	$j = 1,\ldots, 13$.	13	Demand by the j'th industry for land.
(4.5)	l_j	=	$a_{Lj} + q_j$	$j = 1,\ldots, 13$.	13	Demand by the j'th industry for labour.
(4.6)	k_j	=	$a_{Kj} + q_j$	$j = 1,\ldots, 13$.	13	Demand by the j'th industry for capital.
(4.7)	a^D_{ij}	=	$-\eta_i [pt^D_{ij} - (S^D_{ij}\, pt^D_{ij} + S^I_{ij}\, pt^I_{ij})]$	$i = 1,\ldots, 13$, $j = 1,\ldots, 13$.	13^2	Input-output coefficient for usage by the j'th industry of the i'th domestically produced commodity.
(4.8)	a^I_{ij}	=	$-\eta_i [pt^I_{ij} - (S^D_{ij}\, pt^D_{ij} + S^I_{ij}\, pt^I_{ij})]$	$i = 1,\ldots, 13$, $j = 1,\ldots, 13$.	13^2	Input-output coefficient for usage by the j'th industry of the i'th imported commodity.
(4.9)	a_{Hj}	=	$-\rho_j [rh_j - (S_{Hj}\, rh_j + S_{Mj}\, rm)]$	$j = 1,\ldots, 13$.	13	Input-output coefficient for usage by the j'th bundle of its labour-capital bundle.
(4.10)	a_{Mj}	=	$-\rho_j [rm - (S_{Hj}\, rh_j + S_{Mj}\, rm)]$	$j = 1,\ldots, 13$.	13	Input-output coefficient for usage by the j'th industry of land.
(4.11)	rh_j	=	$S_{Wj}\, w + S_{Rj}\, r$	$j = 1,\ldots, 13$.	13	Shadow price of the labour-capital bundle used by the j'th industry.
(4.12)	a_{Lj}	=	$a_{Hj} + -\sigma_j [w - rh_j]$	$j = 1,\ldots, 13$.	13	Input-output coefficient for usage by the j'th industry of labour.
(4.13)	a_{Kj}	=	$a_{Hj} + -\sigma_j [r - rh_j]$	$j = 1,\ldots, 13$.	13	Input-output coefficient for usage by the j'th industry of capital.
(4.14)	inv^D_i	=	inv^T_R	$i = 1,\ldots, 13$.	13	Investment usage of the i'th domestically produced commodity.
(4.15)	inv^I_i	=	inv^T_R	$i = 1,\ldots, 13$.	13	Investment usage of the i'th imported commodity.

(Continued)

Identifier	Equation	Subscript range	Number	Description
(4.16)	$c_g = \mu_{Cg}\, yd + \sum_h \lambda_{gh}\, p_{Cg}$	g = 1,..., 10.	10	Demand for the g'th consumer commodity.
(4.17)	$sv = \mu_S\, yd + \sum_g \lambda_{Sg}\, p_{Cg}$		1	Supply of private savings.
(4.18)	$p_{Cg} = S^D_{Cg}\, p^D_{Cg} + S^I_{Cg}\, p^I_{Cg}$	g = 1,..., 10.	10	Shadow price of the g'th consumer commodity.
(4.19)	$c^D_g = c_g - \beta_g \left[p^D_{Cg} - p_{Cg}\right]$	g = 1,..., 10.	10	Demand for the g'th domestically produced consumer commodity.
(4.20)	$c^I_g = c_g - \beta_g \left[p^I_{Cg} - p_{Cg}\right]$	g = 1,..., 10.	10	Demand for the g'th imported consumer commodity.
(4.21)	$pc^D_i = \sum_g z^D_{(C)ig}\, c^D_g$	g = 1,..., 13.	13	Household consumption of the i'th domestically produced commodity.
(4.22)	$pc^I_i = \sum_g z^I_{(C)ig}\, c^I_g$	g = 1,..., 13.	13	Household consumption of the i'th imported commodity.
(4.23)	$e_i = \epsilon_i \left(pe_i - er - p_{Wi}\right)$	i = 1,..., 13.	13	Exports of the i'th commodity.
(4.24)	$g = \left[z_G - zpi\right] - \psi \left[p_G - zpi\right]$		1	Total real government usage of production commodities.
(4.25)	$l_G = \left[z_G - zpi\right] - \Psi \left[w - zpi\right]$		1	Government usage of labour.
(4.26)	$k_G = \left[z_G - zpi\right] - \Psi \left[r - zpi\right]$		1	Government usage of capital.
(4.27)	$p_G = \sum_i S^D_{Gi}\, p^D_i + \sum_i S^I_{Gi}\, p^I_i + \sum_i \left(S^D_{Gi} + S^I_{Gi}\right) t_{Gi}$		1	Price index for government usage of production commodities.
(4.28)	$gov^D_i = g$	i = 1,..., 13.	13	Government usage of the i'th domestically produced commodity.

(Continued)

Identifier	Equation	Subscript range	Number	Description
(4.29)	$gov_i^I = g$	$i = 1,\ldots, 13.$	13	Government usage of the i'th imported commodity.
(4.30)	$dstoc_i^D = f_S + h_S q_i$	$i = 1,\ldots, 13.$	13	Change in stocks of th i'th domestically produced commodity.
(4.31)	$dstoc_i^I = f_S + h_S imp_i$	$i = 1,\ldots, 13.$	13	Change in stocks of the i'th imported commodity.
(4.32)	$p_j^D = -s_{Qj} + \sum_i H^D_{(X)ij}\, pt^D_{ij} + \sum_i H^I_{(X)ij}\, pt^I_{ij} + H_{Lj} w + H_{Kj} r + H_{Mj}\, rm$	$j = 1,\ldots, 13.$	13	Zero profits condition for the j'th industry.
(4.33)	$p_i^I = d_i + er + p_{Wi}$	$i = 1,\ldots, 13.$	13	Basic price of the i'th imported commodity.
(4.34)	$pt^D_{ij} = t^D_{ij} + p^D_i$	$i = 1,\ldots, 13,$ $j = 1,\ldots, 13.$	13^2	Price paid by the j'th industry for the i'th domestically produced commodity.
(4.35)	$pt^I_{ij} = t^I_{ij} + p^I_i$	$i = 1,\ldots, 13,$ $j = 1,\ldots, 13.$	13^2	Price paid by the j'th industry for the i'th imported commodity.
(4.36)	$p^D_{Cg} = \sum_i S^D_{(C)ig} (p^D_i + t^D_{Ci})$	$g = 1,\ldots, 10.$	10	Price paid by households for the g'th domestically produced consumer commodity.
(4.37)	$p^I_{Cg} = \sum_i S^I_{(C)ig} (p^I_i + t^I_{Ci})$	$g = 1,\ldots, 10$	10	Price paid by households for the g'th imported consumer commodity.
(4.38)	$p_{Ei} = t_{Ei} + p^D_i$	$i = 1,\ldots, 13$	13	Fob price of exports of the i'th commodity.
(4.39)	$q_i = \sum_j S^D_{(X)ij}\, x^D_{ij} + S^D_{INVi}\, inv^D_i + S^D_{PCi}\, pc^D_i + S^D_{Ei}\, e_i + S^D_{GOVi}\, gov^D_i + S^D_{DSTOCi}\, dstoc^D_i$	$i = 1,\ldots, 13$	13	Market clearing condition for the i'th domestically produced commodity.
(4.40)	$imp_i = \sum_j S^I_{(X)ij}\, x^I_{ij} + S^I_{INVi}\, inv^I_i + S^I_{PCi}\, pc^I_i + S^I_{GOVi}\, gov^I_i + S^I_{DSTOCi}\, dstoc^I_i$	$i = 1,\ldots, 13.$	13	Imports of the i'th commodity.

(Continued)

(4.41)	l^T	=	$\sum_j S_{LTJ}\, l_j + S_{LTG}\, l_G$	1	Aggregate usage of labour.
(4.42)	k^T	=	$\sum_j S_{KTJ}\, k_j + S_{KTG}\, k_G$	1	Aggregate usage of capital.
(4.43)	m^T	=	$\sum_j S_{MTJ}\, m_j$	1	Aggregate usage of land.
(4.44)	inv^T	=	$p_I + inv_R^T$	1	Aggregate investment at current prices.
(4.45)	p_I	=	$\sum_i S^D_{(INVT)i}\, p^D_i + \sum_i S^I_{(INVT)i}\, p^I_i$ $+ \sum_i (S^D_{(INVT)i} + S^I_{(INVT)i})\, t_{Ii}$	1	Purchase price of capital.
(4.46)	c^T	=	$\sum_g S^D_{(CT)g}\, (c^D_g + p^D_{Cg})$ $+ \sum_g S^I_{(CT)g}\, (c^I_g + p^I_{Cg})$	1	Aggregate consumption at current prices.
(4.47)	cpi	=	$\sum_g S^D_{(CT)g}\, p^D_{Ci} + \sum_g S^I_{(CT)g}\, p^I_{Ci}$	1	Consumer price index.
(4.48)	c_R^T	=	c^T - cpi	1	Real aggregate consumption.
(4.49)	e^T	=	$\sum_i S_{(ET)i}\, (p_{Ei} + e_i)$	1	Aggregate exports at current FOB prices.
(4.50)	epi	=	$\sum_i S_{(ET)i}\, p_{Ei}$	1	Export price index (FOB prices).
(4.51)	e_R^T	=	e^T - epi	1	Real aggregate exports.
(4.52)	$dstoc^T$	=	$\sum_i S^D_{(ST)i}\, (t_{si} + p^D_i + dstoc^D_i)$ $+ \sum_i S^I_{(ST)i}\, (t_{si} + p^I_i + dstoc^I_i) + r_s$	1	Aggregate change in stocks at current prices.
(4.53)	dstocpi	=	$\sum_i S^D_{(ST)i}\, p^D_i + \sum_i S^I_{(ST)i}\, p^I_i + \sum_i (S^D_{(ST)i} + S^I_{(ST)i})\, t_{si}$	1	Price index for change in stocks.
(4.54)	$dstoc_R^T$	=	$dstoc^T$ - dstocpi	1	Real change in stocks.
(4.55)	imp^T	=	$\sum_i S_{(IT)i}\, (p_{Wi} + imp_i)$	1	Aggregate imports at current foreign currency prices.

(Continued)

Identifier	Equation			Subscript range	Number	Description
(4.56)	imppi	=	$\sum_i S_{(IT)i}\, p_i^I$		1	Import price index.
(4.57)	imp_R^T	=	imp^T - imppi		1	Real aggregate imports.
(4.58)	100.delb	=	E. e^T - IMP (er + imp^T)		1	Balance of trade, at current domestic prices.
(4.59)	$100.delb^R$	=	100.delb - BT . imppi		1	Real balance of trade.
(4.60)	delrbt	=	(100/NDPE).delb - (BT/NDPE).ndpe		1	Ratio of the balance of trade to NDP from the expenditure side.
(4.61)	depr	=	ζ + k + p_I		1	Depreciation of the capital stock.
(4.62)	zpi	=	$S_{GZ}\, p_G + S_{LZ}\, w + S_{KZ}\, r$		1	Price index for government spending.
(4.63)	z_{GR}	=	z_G - zpi		1	Real government spending.
(4.64)	z_G	=	$c^T + f_Z$		1	Government spending relative to private consumption spending.
(4.65)	t_{GR}	=	t_G - cpi		1	Real transfer payments by government.
(4.66)	t_G	=	$y_{FN} + f_{TG}$		1	Transfer payments by government, relative to net factor income.
(4.67)	opi	=	S_{SO} zpi + S_{TO} cpi		1	Price index for government outlays.
(4.68)	r_G	=	$\sum_i S_{(DR)i} (er + p_{Wi} + imp_i + R_{(IMP)i}\, d_i)$ + $\sum_i \sum_j S^D_{(TR)ij} (x^D_{ij} + p^D_i + R^D_{(INT)ij}\, t^D_{ij})$			

(Continued)

			$+ \sum_i \sum_j S^I_{(TR)ij} \left(x^I_{ij} + p^I_i + R^I_{(INT)ij}\, t^I_{ij}\right)$		
			$+ \sum_i S^D_{(CR)i} \left(pc^D_i + p^D_i + R^D_{(CON)i}\, t^D_{Ci}\right)$		
			$+ \sum_i S^I_{(CR)i} \left(pc^I_i + p^I_i + R^I_{(CON)i}\, t^I_{Ci}\right)$		
			$+ \sum_i S^D_{(IR)i} \left(inv^D_i + p^D_i + R_{(INV)i}\, t_{Ii}\right)$		
			$+ \sum_i S^I_{(IR)i} \left(inv^I_i + p^I_i + R_{(INV)i}\, t_{Ii}\right)$		
			$+ \sum_i S_{(ER)i} \left(e_i + p^D_i + R_{(EXP)i}\, t_{Ei}\right)$		
			$+ \sum_i S^D_{(GR)i} \left(gov^D_i + p^D_i + R_{(GOV)i}\, t_{Gi}\right)$		
			$+ \sum_i S^I_{(GR)i} \left(gov^I_i + p^I_i + R_{(GOV)i}\, t_{Gi}\right)$		
			$+ \sum_i S^D_{(STR)i} \left(dstoc^D_i + p^D_i + R_{(STOC)i}\, t_{Si}\right)$		
			$+ \sum_i S^I_{(STR)i} \left(dstoc^I_i + p^I_i + R_{(STOC)i}\, t_{Si}\right)$		
			$- \sum_j S_{(SR)j} \left(q_j + p^D_j + R_{(SUB)j}\, s_{Qj}\right)$		
			$+ R_{YR} \left(t_Y + y\right) + h_{T1} R_{TAUR}\, cpi + h_{T2}\, R_{TAUR}\, y$	1	Total government revenue.
(4.69)	r_{GR}	=	$r_G - opi$	1	Real government revenue.
(4.70)	100 delbd	=	REV. r_G - OUT.$(z_G + t_G)$	1	Government budget deficit.
(4.71)	100 delbdr	=	100 delbd - BD.opi	1	Real budget deficit.

(Continued)

Identifier	Equation			Subscript range	Number	Description
(4.72)	delrbd	=	(100/NDPE).delbd - (BD/NDPE).ndpe		1	Ratio of the budget deficit to NDP from the expenditure side.
(4.73)	y_{FN}	=	S_{LYF} (w + l) + S_{KYF} (r + k) + S_{MYF} (rm + m) - S_{DYF} depr		1	Aggregate factor income.
(4.74)	y	=	S_{YFY} y_{FN} + S_{TY} t_G		1	Aggregate private income.
(4.75)	yd	=	$(R_{YYD} - R_{YD})$ Y - R_{YD} t_Y - $h_{T1}R_{TYD}$ cpi - h_{T2} $R_{TYD}y$		1	Aggregate household disposable income.
(4.76)	y_R^d	=	yd - cpi		1	Real aggregate household disposable income.
(4.77)	gdppd	=	S_{LGDP} w + S_{KGDP} r + S_{MGDP}rm		1	GDP deflator at factor cost.
(4.78)	ndp_E	=	S_{CN} c^T + S_{IN} inv^T - S_{DN} $depr^T$ + S_{ZN} z_G + S_{EN} e^T - S_{IMPN} $(er + imp^T)$ + S_{DSN} $dstoc^T$		1	Net domestic product from the expenditure side.
(4.79)	ndp_D	=	$S_{CN}c^T$ + S_{SN} sv + S_{RN} r_G - S_{TN} t_G		1	Net domestic product from the disposition side.

(Continued)

Identifier	Equation	Subscript range	Number	Description
(4.80)	$ndppi_E = S_{CN}\ cpi + (S_{IN} + S_{DN})P_I + S_{ZN}\ zpi + S_{EN}\ P_{wi}$ $+ S_{IMPN}\ (er + P_{wi}) + S_{DSN}\ dstocpi$		1	Price index for net domestic product from the expenditure side.
(4.81)	$ndpe_R = ndp_E - ndppi_E$		1	Real net domestic product from the expenditure side.

Equation	Coefficient or parameter	Description
(4.1) - (4.6)	None	
(4.7) - (4.8)	S^D_{ij}	Share of the i'th domestically produced commodity in intermediate usage by the j'th industry of the i'th composite.
	S^I_{ij}	Share of the i'th imported commodity in intermediate usage by the j'th industry of the i'th composite commodity.
(4.9) - (4.10)	S_{Hj}	Share of the labour-capital bundle in value added in the j'th industry.
	S_{Mj}	Share of land in value added in the j'th industry.
(4.11) - (4.13)	S_{Wj}	Share of labour in the labour-capital bundle in the j'th industry.
	S_{Rj}	Share of capital in the labour-capital bundle in the j'th industry.
(4.14)-(4.15)	None	
(4.16)	μ_{Cg}	Income elasticity of demand for the g'th consumer commodity.
	λ_{gn}	Price elasticity of demand for the g'th consumer commodity.
(4.17)	μ_S	Elasticity of savings with respect to disposable income.
	λ_{Sg}	Elasticity of savings with respect to the price of the g'th consumer commodity.
(4.18)-(4.20)	S^D_{Cg}	Share of the domestically produced variety in expenditure on the g'th consumer composite commodity.
	S^I_{Cg}	Share of the g'th imported variety in expenditure on the g'th consumer commodity.
(4.21)	$Z^D_{(C)ig}$	Share of the g'th domestically produced consumer commodity in consumption usage of the i'th domestically producer commodity.
(4.22)	$Z^I_{(C)ig}$	Share of the g'th imported consumer commodity in consumption usage of the i'th imported producer commodity.
(4.23)	None	
(4.24)-(4.26)	S_{GZ}	Share of goods and services in government spending.
	S_{LZ}	Share of labour in government spending.
	S_{KZ}	Share of capital in government spending.
(4.27)	S^D_{Gi}	Share of the i'th domestically produced commodity in total government expenditure on goods and services.
	S^I_{Gi}	Share of government expenditure on the i'th imported commodity in total government expenditure on goods and services.
(4.28)-(4.29)	None	
(4.30)-(4.31)	h_S	Indexing parameter for nominal stocks
(4.32)	$H^D_{(X)ij}$	Share of the i'th domestically produced commodity in the total cost of industry j.
	$H^I_{(X)ij}$	Share of the i'th imported commodity in the total cost of industry j.
	H_{Lj}	Share of labour in the total cost of industry j.
	H_{Kj}	Share of capital in the total cost of industry j.
	H_{Mj}	Share of land in the total cost of industry j.
(4.33)-(4.35)	None	
(4.36)	$S^D_{(C)ig}$	Share of the i'th domestically produced commodity in household expenditure on the g'th domestically produced consumer commodity.
(4.37)	$S^I_{(C)ig}$	Share of the i'th imported commodity in household expenditure on the g'th imported consumer commodity.

continued

Equation	Coefficient or parameter	Description
(4.38)	None	
(4.39)	$S^{D}_{(X)ij}$	Share of intermediate usage by the j'th industry in output of the i'th domestically produced commodity.
	S^{D}_{INVi}	Share of investment usage in output of the i'th domestically produced commodity.
	S^{D}_{PCi}	Share of consumption usage in output of the i'th domestically produced commodity.
	S^{D}_{Ei}	Share of exports in output of the i'th domestically produced commodity.
	S^{D}_{GOVi}	Share of government usage in output of the i'th domestically produced commodity.
	S^{D}_{DSTOCi}	Share of change in stocks in output of the i'th domestically produced commodity.
(4.40)	$S^{I}_{(X)ij}$	Share of intermediate usage by the j'th industry in imports of the i'th commodity.
	S^{I}_{INVi}	Share of investment usage in imports of the i'th commodity.
	S^{I}_{PCi}	Share of government usage in imports of the i'th commodity.
	S^{I}_{GOVi}	Share of government usage in imports of the i'th commodity.
	S^{I}_{DSTOCi}	Share of change in stocks in imports of the i'th commodity.
(4.41)	S_{LTj}	Share of labour used in industry j in total labour usage.
	S_{LTg}	Share of labour used by government in total labour usage.
(4.42)	S_{KTj}	Share of capital used in industry j in total capital usage.
	S_{KTG}	Share of capital used by government in total capital usage.
(4.43)	S_{MTj}	Share of land used by industry j in total land usage.
(4.44)	None	
(4.45)	$S^{D}_{(INVT)i}$	Share of the i'th domestically produced commodity in investment expenditure.
	$S^{I}_{(INVT)i}$	Share of the i'th imported commodity in investment expenditure.
(4.46)	$S^{D}_{(CT)g}$	Share of the g'th domestically produced consumer commodity in aggregate consumption.
	$S^{I}_{(CT)g}$	Share of of the g'th imported consumer commodity in aggregate consumption.
(4.47)	$S^{D}_{(CT)g}$	See (4.47).
	$S^{I}_{(CT)g}$	See (4.47).
(4.48)	None	
(4.49)	$S_{(ET)i}$	Share of exports of the i'th commodity in total exports at FOB prices.
(4.50)	$S_{(ET)i}$	See (4.49).
(4.51)	None	
(4.52)	$S^{D}_{(ST)i}$	Share of the i'th domestically produced commodity in aggregate change in stocks.
	$S^{I}_{(ST)i}$	Share of the i'th imported commodity in aggregate change in stocks.
(4.53)	$S^{D}_{(ST)i}$	See (4.52).
	$S^{I}_{(ST)i}$	See (4.52).
(4.54)	None	
(4.55)	$S_{(IT)i}$	Share of the i'th imported commodity in aggregate imports at foreign currency prices.
(4.56)	$S_{(IT)i}$	See (4.55).
(4.57)	None	

continued

Equation	Coefficient or parameter	Description
(4.58)	E	Value of total exports at current domestic prices.
	IMP	Value of total imports at current domestic prices.
(4.59)	BT	Balance of trade.
(4.60)	BT	See (4.59).
	NDPE	Net domestic product from the expenditure side.
(4.61)	None	
(4.62)	S_{GZ}	Share of government spending on producer commodities in total government spending.
	S_{LZ}	Share of government spending on labour in total government spending.
	S_{KZ}	Share of government spending on capital in total government spending.
(4.63)	None	
(4.64)	None	
(4.65)	None	
(4.66)	None	
(4.67)	S_{SO}	Share of government spending in government outlays.
	S_{TO}	Share of government transfer payments in government outlays.
(4.68)	$S_{(DR)i}$	Share of tariff revenue on commodity i in total government revenue.
	$S^{D}_{(TR)ij}$	Share of commodity tax revenue on the usage of the i'th domestically produced commodity in the j'th industry in total government revenue.
	$S^{I}_{(TR)ij}$	Share of commodity tax revenue on the usage of the i'th imported commodity in the j'th industry in total government revenue.
	$S^{D}_{(CR)i}$	Share of commodity tax revenue on household consumption of the i'th domestically produced commodity in total government revenue.
	$S^{I}_{(CR)i}$	Share of commodity tax revenue on household consumption of the 'th imported commodity in total government revenue.
	$S^{D}_{(IR)i}$	Share of commodity tax revenue on investment usage of the i'th domestically produced commodity in total government revenue.
	$S^{I}_{(IR)i}$	Share of commodity tax revenue on investment usage of the i'th imported commodity in total government revenue.
	$S_{(ER)i}$	Share of commodity tax revenue on exports of the i'th commodity in total government revenue.
	$S^{D}_{(GR)i}$	Share of commodity tax revenue on government usage of the i'th domestically produced commodity in total government revenue.
	$S^{I}_{(GR)i}$	Share of commodity tax revenue on government usage of the i'th imported commodity in total government revenue.
	$S^{D}_{(STR)i}$	Share of commodity tax revenue on the i'th domestically produced commodity used as a stock in total government revenue.
	$S^{I}_{(STR)i}$	Share of commodity tax revenue on the i'th imported commodity used as a stock in total government revenue.
	$S_{(SR)j}$	Share of subsidies on output of the j'th industry in total government revenue.
	$R_{(IMP)i}$	Ratio of the power of tariff rate of i'th import to the ad valorem rate.
	$R^{D}_{(INT)ij}$	Ratio of the power of the commodity tax rate on usage of the j'th industry of the i'th domestically produced commodity to the ad valorem rate.
	$R^{I}_{(INT)ij}$	Ratio of the power of the commodity tax rate on usage of the j'th industry of the i'th imported commodity to the ad valorem rate.
	$R_{(INV)i}$	Ratio of the power of the commodity tax rate on investment usage of the i'th commodity to the ad valorem rate.
	$R^{D}_{(CON)i}$	Ratio of the power of the commodity tax rate on consumption of the i'th domestically produced commodity to the ad valorem rate.

continued

Equation	Coefficient or parameter	Description
	$R^{r}_{(CON)i}$	Ratio of the power of the commodity tax rate on consumption of the commodity tax rate on consumption of the i'th imported commodity to the ad valorem rate.
	$R_{(EXP)i}$	Ratio of the power of the commodity tax rate on exports of the i'th commodity to the ad valorem rate.
	$R_{(GOV)i}$	Ratio of the power of the commodity tax rate on government usage of the i'th commodity to the ad valorem rate.
	$R_{(STOC)i}$	Ratio of the power of the commodity tax rate on change in stocks of the i'th commodity to the ad valorem rate.
	$R_{(SUB)j}$	Ratio of the power of the net subsidy rate on the j'th industry to the ad valorem rate.
	R_{YR}	Ratio of the multiple of the marginal tax rate by household income to total government revenue.
	R_{TAVR}	Ratio of intercept of the linear personal income tax schedule to total government revenue.
	h_{T1}	Parameter indexing the intercept of the linear personal income tax schedule to the CPI.
	h_{T2}	Parameter indexing the intercept of the linear personal income tax schedule to private income.
(4.69)	None	
(4.70)	REV	Total government revenue.
	OUT	Total government outlays.
(4.71)	BD	Government budget deficit.
(4.72)	BD	See (4.71)
	NDPE	See (4.60)
(4.73)	S_{LYF}	Share of labour income in aggregate factor income.
	S_{KYF}	Share of capital income in aggregate factor income.
	S_{MYF}	Share of income from land in aggregate factor income.
	S_{DYF}	Share of depreciation of capital in aggregate factor income.
(4.74)	S_{YFY}	Share of aggregate factor income in aggregate household income.
	S_{TY}	Share of income from transfers by government in aggregate household income.
(4.75)	R_{YYD}	Ratio of household income to household disposable income.
	R_{YD}	Ratio of the multiple of the marginal tax rate by household income to household disposable income.
	R_{TYD}	Ratio of intercept in the linear personal income tax schedule to aggregate household disposable income.
	h_{T1}	See (4.68).
	h_{T2}	See (4.68).
(4.76)	None	
(4.77)	S_{LGDP}	Share of labour in GDP at factor cost.
	S_{KGDP}	Share of capital in GDP at factor cost.
	S_{MGDP}	Share of land in GDP at factor cost.
(4.78)	S_{CN}	Share of household consumption in net domestic product.
	S_{IN}	Share of investment in net domestic product.
	S_{DN}	Share of depreciation on capital in net domestic product.

continued

Equation	Coefficient or parameter	Description
	S_{ZN}	Share of government spending in net domestic product.
	S_{EN}	Share of exports in net domestic product.
	S_{IMPN}	Share of imports at domestic prices in net domestic product.
	S_{DSN}	Share of stocks in net domestic product.
(4.79)	S_{CN}	See (4.72).
	S_{SN}	Share of savings in net domestic product.
	S_{RN}	Share of government revenue in net domestic product.
	S_{TN}	Share of government transfer payments in net domestic product.
(4.80)	S_{CN}	see (4.71).
	S_{IN}	see (4.71).
	S_{DN}	see (4.71).
	S_{ZN}	see (4.71).
	S_{EN}	see (4.71).
	S_{IMPN}	see (4.71).
	S_{DSN}	see (4.71).
(4.81)	None	

TABLE 6 : A TYPICAL LIST OF EXOGENOUS VARIABLES FOR THE [illegible]

Variable	Subscript range[a]	Number[a]	Description
T^D_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Commodity tax rate on usage by the j'th industry of the i'th domestically produced commodity.
T^I_{ij}	$i = 1,\ldots,n_s$, $j = 1,\ldots,n_s$.	n_s^2	Commodity tax rate on usage by the j'th industry of the i'th imported commodity.
D_i	$i = 1,\ldots,n_s$.	n_s	Tariff rate on imports of the i'th commodity.
P_{Wi}	$i = 1,\ldots,n_s$.	n_s	World price of the i'th commodity.
T^D_{Ci}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on consumption of the i'th domestically produced commodity.
T^I_{Ci}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on consumption of the i'th imported commodity.
T_{Ei}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on exports of the i'th commodity.
T_{Gi}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on government usage of the i'th commodity.
T_{Ii}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on investment usage of the i'th commodity.
T_{Si}	$i = 1,\ldots,n_s$.	n_s	Commodity tax rate on change in stocks of the i'th commodity.
S_{Qj}	$j = 1,\ldots,n_s$.	n_s	Production subsidy rate on output of the j'th industry.
$DSTOC^T_R$		1	Aggregate real change in stocks.
ER		1	Exchange rate.
INV^T_R		1	Aggregate gross real investment.
K^T		1	Aggregate usage of capital.
L^T		1	Aggregate usage of labour.
M^T		1	Aggregate usage of land.
T_G		1	Transfer payments by government.
T_Y		1	Marginal income tax rate.
Z_G		1	Government spending.

a n_s denotes the number of sectors in the model. n_c denotes the number of consumer commodities.

Vector Variable	Dimension[a]	Typical element	Description
t^D	n_s^2	t^D_{ij}	Powers of taxes on intermediate usage of domestically produced commodities.
t^I	n_s^2	t^I_{ij}	Powers of taxes on intermediate usage of imported commodities.
pc^D	n_s	pc^D_i	Consumption of domestically produced producer commodities.
pc^I	n_s	pc^I_i	Consumption of imported producer commodities.
d	n_s	d_i	Powers of import tariffs.
e	n_s	e_i	Export volumes.
imp	n_s	imp_i	Import volumes.
p^D	n_s	p^D_i	Basic prices of domestically produced commodities.
p^I	n_s	p^I_i	Basic prices of imported commodities.
p_w	n_s	p_{wi}	World prices.
t^D_c	n_s	t^D_{Ci}	Powers of taxes on consumption of domestically produced commodities.
t^I_c	n_s	t^I_{Ci}	Powers of taxes on consumption of imported commodities.
t_E	n_s	t_{Ei}	Powers of taxes on exports.
t_G	n_s	t_{Gi}	Powers of taxes on government usage of producer commodities.
t_I	n_s	t_{Ii}	Powers of taxes on investment usage.
t_S	n_s	t_{Si}	Powers of taxes on changes in stocks.
k	n_s	k_j	Capital usage by industry.
l	n_s	l_j	Labour usage by industry.
m	n_s	m_j	Land usage by industry.
q	n_s	q_j	Industry output levels.
s_Q	n_s	s_{Qj}	Powers of production subsidies.
c^D	n_c	c^D_g	Consumption of domestically produced consumer commodities.
c^I	n_c	c^I_g	Consumption of imported consumer commodities.
$deld_d$	1	-	Change in budget deficit.
$deld_R$	1	-	Change in real budget deficit.
delb	1	-	Change in balance of trade.
$delb_R$	1	-	Change in real balance of trade.
c^T	1	-	Aggregate consumption expenditure.
c^T_R	1	-	Real aggregate consumption.
cpi	1	-	Consumer price index.
depr	1	-	Depreciation of capital.
$dstoc^T_R$	1	-	Real change in stocks.
$dels^T_R$	1	-	Change in real inventory investment.
dstocpi	1	-	Price index for change in stocks.
e^T	1	-	Aggregate export revenue.
e^T_R	1	-	Real aggregate exports.
epi	1	-	Export price index.
er	1	-	Exchange reate (units of domestic currency for unit of foreign currency).
g	1	-	Real aggregate government usage of producer commodies.

(Continued)

TABLE 7 : VECTOR VARIABLES IN THE WALRAS CONDENSED SYSTEM (Continued)

Vector Variable	Dimension[a]	Typical element	Description
$gdppi_{FC}$	1	-	Price index for GDP at factor cost.
imp^T	1	-	Aggregate expenditure on imports (foreign currency units).
imp_R^T	1	-	Aggregate real imports.
imppi	1	-	Import price index (foreign currency units).
inv^T	1	-	Aggregate investment expenditure.
inv_R^T	1	-	Real aggregate investment.
k^T	1	-	Aggregate usage of capital.
k_G	1	-	Government usage of capital.
l^T	1	-	Aggregate usage of labour.
l_G	1	-	Government usage of labour.
m^T	1	-	Aggregate usage of land.
ndp_D	1	-	NDP (calculated from the disposition side).
ndp_E	1	-	NDP (calculated from the expenditure side).
ndp_{ER}	1	-	Real NDP (calculated from the expenditure side).
$ndppi_E$	1	-	Price index for NDP (calculated from the expenditure side).
opi	1	-	Price index for government outlays.
p_G	1	-	Price index for government usage of producer commodities.
p_I	1	-	Purchase price of capital.
r	1	-	Rental price of capital.
r_G	1	-	Government revenue.
r_{GR}	1	-	Real government revenue.
rm	1	-	Rental price of land.
sv	1	-	Private saving.
t_G	1	-	Transfer payments by government.
t_{GR}	1	-	Real transfer payments by government.
t_y	1	-	Marginal income tax rate.
w	1	-	Wage rate.
y	1	-	Aggregate household income.
yd	1	-	Aggregate disposable income.
yd_R	1	-	Real aggregate disposable income.
z_G	1	-	Government spending.
z_{GR}	1	-	Real government spending.
zpi	1	-	Price index for government spending.

a n_s denotes the number of sectors in the model. n_c denotes the number of consumer commodities.

TABLE 8 : CONTENTS OF THE INPUT-OUTPUT DATA FILE FOR THE WALRAS MODEL OF THE AUSTRALIAN ECONOMY

Header	Type[a]	Dimensions[b]	Description
AI02	R	$n_s \times n_s$	Intermediate usage (domestic)
AI04	R	$n_s \times n_s$	Intermediate usage (imported)
AI06	R	$n_s \times 1$	Investment (domestic)
AI08	R	$n_s \times 1$	Investment (imported)
AI10	R	$n_s \times n_c$	Consumption (domestic)
AI12	R	$n_s \times n_c$	Consumption (imported)
AI14	R	$n_s \times 1$	Exports
AI16	R	$n_s \times 1$	Government usage (domestic)
AI18	R	$n_s \times 1$	Government usage (imported)
AI20	R	$n_s \times 1$	Change in stocks (domestic)
AI22	R	$n_s \times 1$	Change in stocks (imported)
AI24	R	$1 \times n_s$	Non-commodity indirect taxes
AI26	R	$1 \times n_s$	Usage of labour
AI28	R	$1 \times n_s$	Usage of capital
AI30	R	$1 \times n_s$	Usage of land
AI32	R	$n_s \times n_s$	Commodity tax on intermediate usage (domestic)
AI34	R	$n_s \times n_s$	Commodity tax on intermediate usage (imported)
AI36	R	$n_s \times 1$	Commodity tax on investment
AI38	R	$n_s \times 1$	Commodity tax on consumption (domestic)
AI40	R	$n_s \times 1$	Commodity tax on consumption (imported)
AI42	R	$n_s \times 1$	Commodity tax on exports
AI44	R	$n_s \times 1$	Commodity tax on government usage
AI46	R	$n_s \times 1$	Commodity tax on change in stocks
AI48	R	$n_s \times 1$	Duty

(Continued)

TABLE 8 : CONTENTS OF THE INPUT-OUTPUT DATA FILE FOR THE WALRAS MODEL OF THE AUSTRALIAN ECONOMY (Continued)

Header	Type[a]	Dimensions[b]	Description
AI50	R	1 x 1	Government usage of labour
AI52	R	1 x 1	Government usage of capital
AI54	R	1 x 1	Income tax
AI56	R	1 x 1	Transfer payments
AI58	R	1 x 1	Capital stock
XXCR	C		Creation information
XXHS	C		History

a R denotes a real number array. c denotes a character array

b n_s denotes the number of sectors in the model. n_c denotes the number of consumer commodities.

TABLE 9 : CONTENTS OF THE PARAMETERS FILE FOR THE WALRAS MODEL OF THE AUSTRALIAN ECONOMY

Header	Type[a]	Dimensions[b]	Description
AP02	R	n_s x 1	Elasticity of substitution between domestic and imported producer commodities
AP04	R	n_s x 1	Elasticity of substitution between the labour-capital bundle and land
AP06	R	n_s x 1	Elasticity of substitution between labour and capital
AP08	R	n_c x 1	Elasticity of consumer demand with respect to income
AP10	R	n_c x n_c	Elasticity of consumer demand with respect to prices
AP12	R	1 x 1	Elasticity of saving with respect to income
AP14	R	1 x n_c	Elasticity of saving with respect to price
AP16	R	n_c x 1	Elasticity of substitution between domestic and imported consumer commodities
AP18	R	n_s x 1	Export demand elasticity
AP20	R	1 x 1	Elasticity of substitution between commodities, labour and capital in government usage
AP22	R	1 x 1	Real rate of return on capital
AP24	R	1 x 1	Depreciation rate
AP26	R	1 x 1	Marginal tax rate
XXCR	C		Creation information
XXHS	C		History

a R denotes a real number array. c denotes a character array.
b n_s denotes the number of sectors in the model. n_c denotes the number of consumer commodities.

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APPENDIX : AN AMENDMENT TO THE CLOSURE PROPOSED IN OECD (1988a)

In setting aggregate investment exogenously, the closure presented in Table 6 differs from that proposed in OECD (1988a). In the closure proposed there, 'investment is determined residually as the sum of savings, depreciation, change in stocks and the net position of the government and foreign sectors' (OECD (1988a), para 41). This is done by including in the equation system an equation

$$INV^T = SV + DEPR + (R_G - Z_G - T_G) - DSTOC^T - (E^T - ER.\overline{IMP}^T) \quad (2.57)$$

(OECD (1988a), equation (1.56)) (for notation see Table 2). This equation is a national accounting identify which can be derived by equating the two measures of NDP calculated in equations (2.55) and (2.56) of Table 1. So adding equation (2.57) to the equation system in Table 1 is equivalent to adding the equation

$$NDP_E = NDP_D \quad (2.58)$$

It is contended here that equation (2.57) should not be used to determine aggregate investment. Indeed, with a valid data base, it cannot be so used. Provided that the data base satisfies certain conditions, the two measures of NDP are identically equal. Then equation (2.57) is satisfied automatically, for any value of INV^T. That is, equations (2.57) and (2.58) are already implicit in the equation system. The conditions which the data base must satisfy are:

- the data base is balanced, in that the total value of each industry's sales is equal to its total costs, and
- the consumer demand system is properly parameterized, so that the budget constraint, that the sum of aggregate consumption and saving is equal to aggregate disposable income, continues to be satisfied when the model is shocked.

Since the data base should satisfy these conditions, equation (2.57) should not be used to determine aggregate investment.

These propositions can be demonstrated by the following argument. If the data base is balanced, then NDP calculated from the expenditure side is equal to NDP calculated from the income side,

$$NDP_E = NDP_I = YFN + TI,$$

where YFN denotes factor income, net of depreciation, and TI denotes indirect taxes. For aggregate total sales is equal to the sum of aggregate intermediate usage and GDP calculated from the expenditure side; and aggregate total costs is equal to the sum of aggregate intermediate usage and GDP calculated from the income side. But for a balanced data base, total sales is equal to total costs in each industry; and the market clearing equations and zero profits conditions

ensure that this equality is preserved when the model is shocked. So aggregate total sales is equal to aggregate total costs. Therefore, GDP calculated from the expenditure side is equal to GDP calculated from the income side; so NDP calculated from the expenditure side is equal to NDP calculated from the income side,

$$NDP_E = NDP_I,$$

as stated above. Furthermore, if the consumer demand system is properly parameterized, then NDP calculated from the income side is equal to NDP calculated from the disposition side. For (in the notation of Tables 1 and 2)

$$NDP_I = YFN + TI$$

$$= YFN + (R_G - TD)$$ where TD denotes income tax, by equation (2.52) of Table 1

$$= (YFN + T_G - TD) + (R_G - T_G)$$

$$= (Y - TD) + (R_G - T_G)$$ by equation (2.53)

$$= YD + (R_G - T_G)$$ by equation (2.54)

$$= C^T + SV + (R_G - T_G)$$ provided that the consumer demand system is properly parameterized.

$$= NDP_D.$$

From these equalities it follows that NDP calculated from the expenditure side is equal to NDP calculated from the disposition side,

$$NDP_E = NDP_D.$$

Hence we conclude that equation (2.58) should be satisfied automatically, for any value of INV^T. The same must then be true for equation (2.57).

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