INDUSTRY
COMMISSION

Incorporating International Capital Mobility into SALTER

by

Robert McDougall

SALTER Working Paper No. 21

JUNE 1993

SALTER working papers document work in progress on the development of the SALTER model of the world economy. They are made available to allow scrutiny of the work undertaken but should not be quoted without the permission of the author(s). Comments on the papers would be most welcome.

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CONTENTS

INCORPORATING INTERNATIONAL CAPITAL MOBILITY INTO SALTER

This paper describes the capital mobility extension to the SALTER model of the world economy.

SALTER is a multi-region multi-sector model of the world economy first documented in Jomini *et al.* (1991). It is designed for policy analysis, in particular for analysing trade and industry policy issues in an international context.

SALTER is designed for both long-run and short-run applications. Trade and industry policies are generally directed towards long-run structural objectives. But changes in these policy areas also generate short-run adjustment pressures. SALTER can simulate both the short-run and long-run effects of these policy changes.

One limitation of SALTER has been its inability to model capital accumulation and international capital movements. Over the long run, structural policies influence the allocation of capital not only between industries but also between countries. If implemented on a sufficiently large scale, they may also affect the size of the world capital stock. SALTER has been able to simulate the effects of policy changes on the allocation of capital within regions in the model, but not on the size of the world capital stock, nor on it allocation between regions.

This limitation is serious, since various theoretical and applied studies suggest that these effects may be important. Thus Baldwin (1992) argues that capital accumulation contributes significantly to the effects of trade liberalisation on output. Likewise the Industry Commission (1991) reports ORANI assistance reduction simulations in which this effect is dominant.

The capital mobility extension to SALTER enables the model to simulate these effects. Extensions to the theoretical structure cover capital accumulation, the international allocation of capital, and the international allocation of investment. These extensions are supported by changes to the pre-existing structure, covering national income, external transactions, household income and government budgets.

Section 1 of the paper provides background information on earlier versions of SALTER. Section 2 describes the theoretical approach followed in incorporating international capital mobility. Sections 3 and 4 describe the modifications made to the equation system and database. The extended model is documented more fully in

Jomini *et al.* (forthcoming). An early application of the extended model is given in Dee, Jomini and McDougall (1992).

1 SALTER background

SALTER is a single-period model, expressed as a system of linear equations written mostly in percentage change form.

Since SALTER is a single-period model, it does not track changes in the economy through time. Instead, it compares alternative states of the economy at a single point in time (SALTER is a *comparative-static* model). Simulation results represent not changes through time, but differences between alternative states of the economy at a single point in time.

The model assumes that at the chosen point in time, the economy is in equilibrium. The equilibrium conditions are given in the theoretical structure and closure of the model. The closure used in a simulation may be adjusted according to the desired length of run. For instance, in a short-run simulation industry capital stocks would typically be exogenous in short-run equilibrium; but in a long-run simulation, they would be endogenous.

The underlying theoretical structure of SALTER is a system of non-linear equations. This system is implemented in linearised form. The variables in the linearised equation system represent changes in variables in the underlying non-linear system. Most variables in the implemented system represent percentage changes in underlying variables, but a few represent absolute changes. The absolute change specification is preferred when the underlying variable is liable to assume a value of zero.

A feature of the model which affects some details of the capital mobility extension is the treatment of exchange rates. Since the model contains several regions, the number of bilateral exchange rates is quite large. With sixteen regions, there would be 240 bilateral exchange rates. Rather than incorporate all these into the model, we incorporate just one exchange rate for each region. These rates are the price in each region's currency of some 'neutral' currency. The neutral currency is not specified in the theoretical structure; users may think of it as some non-national unit such as SDRs, or some widely traded national currency such as the US dollar.

2 Overview of the capital mobility extension

The capital mobility extension enables the model to simulate the effects of policy changes on each region's capital stock. To meet this objective, the extension contains modules covering wealth accumulation in each region, the international allocation of capital and the international allocation of investment. It also extends national, household sector and government sector accounts to cover international investment flows and international income payments.

In designing the extension, the basic premise is that capital is internationally mobile. The extended model must therefore include international financial flows, stocks of international assets and liabilities, and international payments of property income.

The remainder of this section presents the framework within which international capital mobility is modelled in the extended model, and then describes how international capital mobility is modelled within this framework.

In setting up the framework, we need to reconsider the treatment of time, and define the new agents, assets, income flows, and tax flows to be included in the model. In modelling international mobility, we need to specify a wealth accumulation process, conditions determining the international allocation of capital and conditions determining the international allocation of investment. We also need to incorporate international property income flows into the national, household sector and government sector accounts.

The framework

Like earlier versions of SALTER, the extended model is comparative static. Nevertheless, in introducing asset accumulation, it is not possible to ignore time entirely. The treatment of time here is based on the assumption that all shocks applied to the model represent discrete changes at a certain initial point in time, while the database and endogenous variables represent values observed at a certain final point (the *terminal instant*). The interval between the initial and terminal instants is the *simulation period*.

As discussed below, stocks of assets are determined in the model by accumulation equations. These equations reflect the intrinsic dynamics of saving and wealth accumulation. Since the model itself is comparative static, the accumulation equations must be derived outside the model, making certain assumptions about the time paths of the explanatory variables in the equations. These assumptions are more or less arbitrary, and may not even be mutually consistent. Unfortunately, eliminating inconsistencies and arbitrary assumptions about time paths would require solving a full multi-period model.

In deriving the accumulation equations, it is necessary to take account of price movements over the simulation period. Recognising price movements raises some questions about the theoretical structure of the model. Changes over time in asset prices generate capital gains or losses, which should perhaps be taken into account in calculating income and saving. Changes over time in exchange rates affect international interest parity conditions, which play a central role in determining the international allocation of capital in the extended model.

Like the original model, the capital mobility extension contains no variables representing time rates of change in prices. We continue to apply at the terminal instant the assumptions in the original model which justify the neglect of capital gain; at the terminal instant, real capital gain is assumed to be zero. We justify a simple form for the interest parity condition, by assuming that at the terminal instant the rate of currency depreciation in each region exactly offsets the domestic rate of inflation. Under this assumption, uncovered interest parity and purchasing power parity imply parity in real rates. These assumptions about the terminal instant are nevertheless compatible with the recognition, in developing the capital accumulation equations, that absolute and relative prices may change through the simulation period.

We add one agent to the model, a representative international financial intermediary.

We add two sets of assets to the model: bonds, and equity in productive assets in each region. Bonds can be owned or owed by each region's representative household, each region's government, or international financial intermediaries. Equity in each region's productive assets is owned entirely by the household sector in that region.

With bonds being sold and bought internationally, some assumption must be made about their denomination. We assume that lenders are protected against inflation by indexation of the principal to a world consumption price index. The index is expressed in terms of the 'neutral' currency (see Section 1). This assumption not only ensures consistency between different regions' borrowings and lendings, but also preserves price homogeneity.

The treatment of agents and assets simplifies reality in two ways. First, it represents all foreign assets and liabilities as debt instruments, whereas in fact they include both debt and equity. Second, it incorporates only net foreign assets, instead of gross assets and gross liabilities. Explaining gross as opposed to net foreign asset ownership would require a more sophisticated approach, accounting for risk and incorporating a theory of portfolio allocation under uncertainty.

As the model incorporates only net asset ownership, so also it incorporates only net property income flows.

The extended model incorporates income taxes on households but not on foreign income recipients (the international financial intermediary). To model these taxes realistically we would need to model gross international income flows. This is because governments do not generally balance taxation of income payments to foreigners with subsidisation of income receipts from foreigners (nor vice versa).

The model separates income taxes on households into taxes on labour income and on property income. It does not separate taxes on household property income into taxes on equity and interest income. To provide a useful separate treatment of taxes on interest income we would need to model gross interest income flows.

International allocation of capital

The international allocation of capital in the extended model is controlled by a set of parity conditions. Equilibrium in international financial markets requires international interest parity. Rate of return maximisation by households imposes parity between the interest rate and the rate of return on equity in each region. The rate of return on equity, together with the price of capital goods, determines the user cost of capital. Together with the user cost of land, it determines the land price.

Because international income flows are not taxed, the international interest parity condition applies to pre-tax interest rates. Because the model applies a common tax rate on household equity income and household interest income, the domestic bond-equity parity conditions can be applied to either pre-tax or post-tax rates.

The parity conditions define the rate of return on capital required by investors. In long-run equilibrium, the actual rate of return on capital is equal to the required rate. But over the short run, actual rates may deviate from required rates. We call the difference between the actual and the required rate the *rate of abnormal return*. The rate of abnormal return can vary between regions and between industries within each region.

Short-run abnormal returns are earned on capital, but not on land. The reason is that the capital goods are produced, while land is not. So the price of capital goods is tied to their replacement cost, while the price of land is free to vary. Land prices accordingly adjust to maintain normal rates of return even over the short run, but capital goods prices do not. So capital typically earns some short-run abnormal return (which may be either positive or negative), but land does not.

The world stock of capital

The total value of world stocks of capital and land in the extended model is equal to total net wealth of households and governments in all regions. This follows from the following premises.

- Household and government net wealth consists of equity in productive assets and net ownership of bonds.
- The value of equity is equal to the value of the underlying productive assets.
- World net ownership of bonds is zero.

Household and government wealth are modelled as the outcome of a wealth accumulation process. We assume that households and governments save or dissave some predetermined fraction of their net disposable income. We also need some assumptions about the behaviour of model variables over the simulation interval. We assume:

— real income grows at a constant rate through the simulation period;

— any change in the saving ratio is concentrated at the beginning of the simulation period; and

— changes in relative prices are concentrated at the beginning of the simulation period.

Under these assumptions we derive *accumulation equations*, expressing wealth at the terminal instant as a function of the saving ratio, real income, and price variables. The derivation of these equations is described in Appendix A.

National, household and government accounts

With the introduction of foreign income flows, a distinction emerges between the value of domestic product and national income. Accordingly, we introduce a variable representing national income, calculated as the sum of domestic factor income, net indirect tax, and net income from abroad (ie. net interest income). To provide a welfare indicator, we also define a variable representing real national income, calculated by deflating national income by a national consumption price index.

We make some changes to the modelling of household income. We introduce a new form of household income, net interest income. We change the income tax base to include household interest income but to exclude transfer receipts from government.

We introduce a new component of government receipts, net interest receipts. Because the new component is net rather than gross interest receipts, we measure total government receipts net of government interest payments. Similarly, we measure total government outlays exclusive of government interest payments.

International allocation of investment

The treatment of the international allocation of investment is based on the same parity conditions as the international allocation of capital. The parity conditions state that capital is allocated across regions so as to equalise rates of return. To maintain this equality over time, investment must be allocated across regions so as to equalise time rates of change in rates of return. This gives a new parity condition, that the expected rate of change in rates of return is uniform across regions and industries. This condition forms the basis of the investment allocation treatment in long-run simulations.

For short-run applications, we recognise that rates of return in individual industries may temporarily deviate from parity. To accommodate this we modify the parity condition for industries earning non-zero abnormal returns. We specify a partial adjustment process under which the expected time rate of change in the rate of abnormal return is proportional and opposite in sign to the rate of abnormal return itself. Investment flows act through this process to eliminate abnormal returns gradually over time.

In a multi-period model, it would be possible to impose model-consistent expectations, requiring rates of return expected to be earned in later periods to be consistent with actual rates in those periods. In SALTER this is not possible, since no later-period results are obtained. In modelling expected rates of return, the most we can aim for is broad consistency with the behaviour of actual rates of return in the model. We choose functional forms and parameter settings in accordance with this objective. The details are set out in Appendix B.

3 Modifications to the SALTER theoretical structure

We follow the convention that original variables in the underlying theory are denoted by upper case letters, and changes or percentage changes in the underlying variables in lower case.

International allocation of capital

We begin by imposing an international interest parity condition:

$$dR_B^z = dR_B \qquad z = 1,\ldots,S \qquad (1)$$

where R_B^z denotes the real rate of return on bonds in region z, and R_B the rate applying to lending and borrowing by the international financial intermediary. We use the absolute rather than the percentage change specification for these variables. This condition reflects profit maximising behaviour in international financial markets.

Next we impose an equity-bond parity condition in each region:

$$dR_E^z = dR_B^z + dF_{RE}^z \qquad z = 1,\ldots,S \qquad (2)$$

where R_E^z denotes the (absolute change in) the normal rate of return on equity in region z, and F_{RE}^z allows for a possible equity premium. With a zero equity premium, this equation reflects profit maximisation by domestic investors; with a non-zero premium, it may be interpreted as reflecting investors' trading off profit against risk.

The next step is to represent the rate of return on fixed capital as the sum of the normal rate of return on equity and a rate of abnormal return:

$$dR_{Kj}^z = dR_E^z + dR_{Aj}^z \qquad j = 1,\ldots,J,\ z = 1,\ldots,S \qquad (3)$$

where dR_{Aj}^z denotes the (absolute change in) the rate of abnormal return on capital in industry j in region z.

The rates of abnormal return would typically be held fixed in long-run simulations, with the capital stocks in each industry in each region free to vary. In short-run simulations the capital stocks would be held fixed, with the rates of abnormal return allowed to vary.

The last step is to use the rate of return variables to relate prices and earnings of productive assets. Here separate treatment is required for capital and land. For capital, we write

$$w_{2j}^{z} = (1 / R_{KGj}^{z})dR_{Kj}^{z} + pci^{z} \qquad j = 1,...,J,\ z = 1,...,S \qquad (4)$$

where w_{2j}^{z} denotes the unit user cost of capital in industry j in region z, pci^{z} denotes the price of capital goods in region z, and R_{KGj}^{z} denotes the gross rate of return on physical capital in industry j in region z. Note that w_{2j}^{z} and pci^{z}, like most subsequent variables, are specified as percentage change variables. The equation is a differential form of the underlying equation:

$$W_{2j}^{z} = (R_{Kj}^{z} + R_{D}^{z})PCI^{z} = R_{KGj}^{z}PCI^{z} \qquad j = 1,...,J,\ z = 1,...,S$$

where R_{D}^{z} denotes the rate of depreciation of capital in region z.

In long-run simulations, equation (4) determines the user cost of capital in each industry in each region. Together with the demand equations for inputs into current production, this determines the inter-industry and inter-regional allocation of capital. In short-run simulations, equation (4) determines the rate of return on capital, including abnormal returns.

For land, we have

$$w_{3}^{z} = (1 / RER^{z})rer^{z} + pn^{z} \qquad z = 1,...,S \qquad (5)$$

where w_{3}^{z} denotes the rental price of land and pn^{z} the purchase price of land in region z, and RER^{z} denotes the normal rate of return on equity, appearing in the linearised form of the equation as a parameter. This equation determines the purchase price of land in each region, in both short-run and long-run simulations.

Equation (4) replaces the following industry-specific returns equation (S70) used in earlier versions of the model:

$$w_{2j}^{z} = w_{2}^{z} + \varphi_{2j}^{z} \qquad j = 1,...,J,\ z = 1,...,S$$

where w_{2}^{z} denotes a general rental price of capital in region z, and φ_{2j}^{z} denotes an industry-specific shift variable. These variables, which appeared only in this equation, are now dropped from the model.

The world capital stock

World net ownership of bonds is equal to the sum over regions of net ownership of bonds in each region:

$$Y.dQ_B^Y + A_B.y = \sum_{z=1}^{S}\left(\frac{Y^z}{E^z} dQ_B^{Yz} + \frac{A_B^z}{E^z} y^z - \frac{A_B^z}{E^z} e^z\right) \qquad (6)$$

where Y denotes world income, Q_B^Y the ratio of world net ownership of bonds to world income, A_B world net ownership of bonds, Y^z income in region z, E^z the exchange rate for region z (relative to the neutral currency), Q_B^{Yz} the ratio of net ownership of bonds to income in region z, and A_B^z net ownership of bonds by region z.

Net ownership of bonds in each region is equal to the sum of net ownership by households and net ownership by governments:

$$Y^z.dQ_B^{Yz} + A_B^z y^z = Y_D^{Hz}.dQ_B^{YHz} + A_B^{Hz}.y_D^{Hz} + R_G^z.dQ_B^{YGz} + A_B^{Gz}.r_G^z \qquad z = 1,\ldots,S \qquad (7)$$

where Y_D^{Hz} denotes household disposable income in region z, Q_B^{YHz} the ratio of net household ownership of bonds to household disposable income in region z, and A_B^{Hz} net ownership of bonds by households in region z; and R_G^z denotes net government receipts in region z, Q_B^{YGz} the ratio of net government ownership of bonds to net government receipts in region z, and A_B^{Gz} net ownership of bonds by governments in region z.

Household wealth is equal to the sum of equity in productive assets and net household ownership of bonds:

$$A^{Hz}.a^{Hz} = A_E^{Hz} a_E^{Hz} + Y_D^{Hz}.dQ_B^{YHz} + A_B^{Hz} y_D^{Hz} \qquad z = 1,\ldots,S \qquad (8)$$

where A^{Hz} denotes household wealth, and A_E^{Hz} equity in productive assets in region z.

Equity in productive assets in each region is equal to the value of productive assets:

$$a_E^{Hz} = S_K^{Az} a_K^z + S_N^{Az} a_N^z \qquad z = 1,\ldots,S \qquad (9)$$

where a_K^z denotes the value of physical capital in region z, and a_N^z the value of land; and S_K^{Az} denotes the share of physical capital in the value of productive assets in region z, and S_N^{Az} the share of land.

In long-run simulations, the variable representing change in the ratio of world net ownership of bonds to world income, dQ_B^Y, is exogenous. This ensures that the initial database condition continues to be met, namely, that the world net ownership of bonds is zero. The change in the ratio of world net ownership of bonds to world income variable is held fixed by allowing the world bond interest rate variable dR_B to vary.

The concept of the short run in this extension is a length of run over which stock variables do not change. The stock accounting in this section and the next ensures that if the stocks of capital and land are held fixed, as they are in a short-run closure, then the world stock of bonds will also be fixed. The initial database condition that world net ownership of bonds is zero is therefore maintained endogenously via the stock accounting. The variable dQ_B^Y no longer needs to be held fixed exogenously to ensure the condition is met. Thus dQ_B^Y is endogenous in the short run, with the world bond interest rate variable dR_B exogenous.

For future use, we define an equity price index for productive assets in each region:

$$p_E^z = S_K^{Az} \cdot pci^z + S_N^{Az} \, pnr^z \qquad z = 1,\ldots,S \qquad (10)$$

where p_E^z denotes the equity price index in region z, and pnr^z the price of land.

The value of physical capital in each region is equal to the product of the price and quantity of physical capital:

$$a_K^z = pci^z + f_{D2}^z \qquad z = 1,\ldots,S \qquad (11)$$

where f_{D2}^z denotes total capital in region z. Similarly, the value of land is equal to the product of the price and quantity of land:

$$a_N^z = pnr^z + f_{D3}^z \qquad z = 1,\ldots,S \qquad (12)$$

where f_{D3}^z denotes land in region z. The equations in this section are sufficient to determine the value of the total world stock of productive assets, given various prices and household and government net wealth in each region. The next section explains how the model determines household and government net wealth.

Wealth accumulation

Household wealth accumulation is given by the equation,

$$\begin{aligned} A^{Hz}a^{Hz} = {} & (A^{Hz} - C_1^{Hz}SV^zT)p_A^{Hz} + C_1^{Hz}Y_D^{Hz}TdQ_S^{YHz} \\ & + C_1^{Hz}SV^zT.cpi^z + C_2^{Hz}SV^zT(y_D^{Hz} - cpi^z) \end{aligned} \qquad z = 1,\ldots,S \qquad (13)$$

where p_A^{Hz} denotes an asset price index for households in region z, Q_S^{YHz} the ratio of household saving to household disposable income in region z, and cpi^z the consumer price index in region z; and SV^z denotes household saving in region z, T the length of the simulation interval, and C_1^{Hz} and C_1^{Hz} are coefficients defined by the formulae

$$C_1^{Hz} = \begin{cases} 1, & \hat{Y}^{Hz}T = 0 \\ \dfrac{1 - e^{-\hat{Y}^{Hz}T}}{\hat{Y}^{Hz}T}, & \text{otherwise} \end{cases}$$

$$C_2^{Hz} = \begin{cases} \dfrac{1}{2}, & \hat{Y}^{Hz}T = 0 \\ \dfrac{\hat{Y}^{Hz}T - 1 + e^{-\hat{Y}^{Hz}T}}{(\hat{Y}^{Hz}T)^2}, & \text{otherwise} \end{cases}$$

where $\hat{Y}^{Hz}$ denotes the rate of growth in real household disposable income over the simulation period.

Each term in equation (13) has the dimensions of wealth in the local region. The term on the left side of the equation represents the change in household nominal wealth, relative to the control solution. The first term on the right side represents that part of the total change in wealth which is due to changes in the prices of assets held at the beginning of the simulation period. The second term on the right side represents the effect of changes in the household saving ratio; the third term, the effects of the nominal component of changes in household income; and the fourth term, the effects of changes in real household income. The coefficients in the third and fourth terms are different because in deriving the accumulation relation, changes in prices relative to the control solution are assumed to be concentrated at the beginning of the simulation period whereas changes in real income are assumed to be occur evenly over the simulation period.

For use in the household wealth accumulation equation, we calculate a household asset price index, using the equation:

$$A^{Hz}p_A^{Hz} = A_E^{Hz}p_E^z + A_B^{Hz}(wcpi + e^z) \qquad z = 1,\ldots,S \qquad (14)$$

where $wcpi$ denotes a world consumption price index, and $wcpi + e^z$ represents that price index converted to the currency of region z. This equation can be derived under the assumption that bonds in all regions are denominated in some common currency, which moves against national currencies in such a way that world inflation, measured in the common currency, is zero.

The calculation of the household asset price index is not strictly correct, since it uses data applying to the end of the simulation period, whereas the index should refer to the situation at the beginning of the simulation period. In practice, this may not lead to major errors in the wealth accumulation equation. In short-run simulations, the composition of household assets is unlikely to change greatly over the simulation period. On the other hand, in simulations where the composition of household assets does change significantly over the forecast period, the terms in the asset accumulation equation which relate to real saving (the second and fourth terms on the right side of equation 13) are likely to dominate the asset revaluation term.

Finally, we have a wealth accumulation relation for government:

$$\begin{aligned} &R_G^z dQ_B^{YGz} + A_B^{Gz} r_G^z \\ &= (A_B^{Gz} - C_1^{Gz}.S^{Gz}T)(wcpi + e^z) + C_1^{Gz}.R_G^z T.dQ_S^{YGz} \\ &+ C_1^{Gz}.S^{Gz}T.zpi^z + C_2^{Gz}.S^{Gz}T(r_G^z - zpi^z) \end{aligned} \qquad z = 1,\ldots,S \qquad (15)$$

where Q_S^{YGz} denotes the ratio of government surplus on current transactions to net government receipts in region z, and zpi^z the government consumption price index in region z; and R_G^z denotes government receipts (net of indirect subsidies and interest payments) in region z, S^{Gz} government surplus on current transactions in region z, and C_1^{Gz} and C_2^{Gz} are coefficients corresponding to C_1^{Hz} and C_1^{Hz}, with the rate of growth in real household disposable income, $\hat{Y}^{Hz}$, replaced by the rate of growth in net government receipts, $\hat{Y}^{Gz}$.

The derivation of the household and government wealth accumulation equations is presented in Appendix A.

National income

National income is equal to the sum of domestic factor income, net income from abroad and net indirect tax. Because the model does not include international flows of labour income, net income from abroad is equivalent to net property income from abroad. Because the model does not include international equity ownership, net property income from abroad is equivalent to net interest income. Thus national income is equal to the sum of domestic factor income, net interest income, and net indirect tax:

$$Y^z y^z = Y_F^z y_F^z + Y^z dQ_I^{Yz} + Y_I^z y^z + R_{GT}^z r_{GT}^z \qquad z = 1,\ldots,S \qquad (16)$$

where Y_F^z denotes domestic factor income, Q_I^{Yz} the ratio of net interest income to national income, Y_I^z net interest income, and R_{GT}^z net indirect tax in region z.

Net interest income in each region is equal to the product of the bond rate and net ownership of bonds:

$$Y^z dQ_I^{Yz} = A_B^z dR_B^z + R_B Y^z dQ_B^{Yz} \qquad z = 1,\ldots,S \qquad (17)$$

National income is also equivalent to the sum of national consumption expenditure and national saving. National consumption expenditure is equal to the sum of household and government consumption expenditure:

$$C_N^z c_N^z = C_T^z c_T^z + Z_G^z z_G^z \qquad z = 1,\ldots,S \qquad (18)$$

where C_N^z denotes national consumption expenditure, C_T^z household consumption expenditure, and Z_G^z government consumption expenditure in region z. Similarly, national saving is equal to the sum of household saving and government surplus on current transactions:

$$Y^z dQ_S^{Yz} + S^z y^z = Y_D^{Hz} dQ_S^{YHz} + SV^z y_D^{Hz} + R_G^z dQ_S^{YGz} + S^{Gz} r_G^z \qquad z=1,\ldots,S \qquad (19)$$

where Q_S^{Yz} denotes the ratio of national saving to national income, and S^z national saving.

We can define real national income as national income deflated by some suitable price index. For the price index, we chose the national consumption price index:

$$V^{Cz} ncpi^z = C_T^z cpi^z + Z_G^z zpi^z \qquad z = 1,\ldots,S \qquad (20)$$

where V^{Cz} denotes national consumption expenditure, C_T^z household consumption expenditure, and Z_G^z government consumption expenditure in region z; and $ncpi^z$ the national consumption price index. Then real national income is given implicitly by the equation

$$y^z = ncpi^z + y_R^z \qquad z = 1,\ldots,S \qquad (21)$$

where y_R^z denotes real national income in region z.

Summing across regions, we can calculate world income:

$$Y.y = \sum_{z=1}^{S} \frac{Y^z}{E^z}(y^z - e^z) \qquad (22)$$

We calculate a world consumption price index as a weighted average of the national consumption price indices for each region, denominated in the neutral currency:

$$V^C.wcpi^z = \sum_{z=1}^{S} \frac{V^{Cz}}{E^z}(ncpi^z - e^z) \qquad z = 1,\ldots,S \qquad (23)$$

where V^C denotes world consumption expenditure (valued in the neutral currency), and $wcpi$ the percentage change in the world price index. Then world real income is given implicitly by the equation:

$$y = wcpi^z + y_R \qquad (24)$$

where y_R denotes world real income.

Household income

The more complete accounting of property income needed to accommodate capital mobility entails extensive changes in the equations relating to household sector income.

Household disposable income is equal to the difference of household income and income tax:

$$Y_D^{Hz} y_D^{Hz} = Y^{Hz} y^{Hz} - R_{GY}^z r_{GY}^z \qquad z = 1,\ldots,S \qquad (25)$$

where Y^{Hz} denotes household income, and R_{GY}^z income tax, in region z. Household income is the sum of labour income, property income, and transfer receipts from government:

$$Y^{Hz} y^{Hz} = Y_L^z y_L^z + Y_P^z y_P^z + T_G^z t_G^z \qquad z = 1,\ldots,S \qquad (26)$$

where Y_L^z denotes labour income, Y_P^z property income, and T_G^z transfer receipts from government in region z.

Labour income is the product of the wage rate and the amount of labour employed:

$$y_L^z = w_1^z + f_{D1}^z \qquad z = 1,\ldots,S \qquad (27)$$

where w_1^z denotes the wage rate in region z, and f_{D1}^z labour employed in region z. Property income is the sum of income from ownership of productive assets and household net interest income:

$$Y_P^z y_P^z = Y_E^z y_E^z + Y_D^{Hz} dQ_I^{YHz} + Y_I^{Hz} y_D^{Hz} \qquad z = 1,\ldots,S \qquad (28)$$

where Y_E^z denotes income from ownership of productive assets, Y_I^{Hz} household net interest income, and Q_I^{YHz} the ratio of household net interest income to household disposable income.

Income from ownership of productive assets is equal to capital earnings plus land earnings less depreciation:

$$Y_E^z y_E^z = \sum_{j=1}^{J} Y_{Kj}^z (w_{2j}^z + f_{2j}^z) + Y_N^z y_N^z - DEP^z dep^z \qquad z = 1,\ldots,S \qquad (29)$$

where Y_{Kj}^z denotes capital earnings in industry j in region z, and Y_N^z land earnings and DEP^z capital depreciation in region z. Household net interest income is equal to the product of the bond rate and household net ownership of bonds:

$$Y_D^{Hz} dQ_I^{YHz} = A_B^{Hz} dR_B^z + R_B Y_D^{Hz} dQ_B^{YHz} \qquad z = 1,\ldots,S \qquad (30)$$

Household consumption expenditure is equal to the product of the household consumption-income ratio and household disposable income. The consumption-income ratio is equal to one minus the saving-income ratio, so

$$C_T^z c_T^z = -Y_D^{Hz} dQ_S^{YHz} + C_T^z y_D^{Hz} \qquad z = 1,\ldots,S \qquad (31)$$

These equations replace equations (S2)-(S5) of Jomini *et al.* (1991), which previously determined household income, disposable income, consumption expenditure and saving. The new treatment of household saving also requires a minor technical change to equation (S7), the equation which calculates net domestic product from the disposition side.

Government receipts and outlays

As with household income, so also with government receipts and outlays, the capital mobility extension requires modifications to the existing equations.

In the extended model, net government receipts are defined as the sum of direct tax, indirect tax, and net interest receipts:

$$R^z r_G^z = R_{GY}^z r_{GY}^z + R_{GT}^z r_{GT}^z + R_G^z dQ_I^{RGz} + R_{GI}^z r_G^z \qquad z = 1,\ldots,S \qquad (32)$$

where R_G^z denotes government receipts, R_{GY}^z income tax, R_{GT}^z indirect tax, R_{GI}^z net government interest receipts, and Q_I^{RGz} the ratio of net government interest receipts to government receipts in region z. Note that government receipts are measured net of indirect subsidies and government interest payments.

Income tax is equal to the sum of tax on labour income and tax on property income:

$$R_{GY}^z r_{GY}^z = R_{GYL}^z r_{GYL}^z + R_{GYP}^z r_{GYP}^z \qquad z = 1,\ldots,S \qquad (33)$$

where R_{GYL}^z denotes tax on labour income, and R_{GYP}^z tax on property income in region z. Tax on labour income is equal to the product of the tax rate and the tax base:

$$r_{GYL}^z = t_{YL}^z + y_L^z \qquad z = 1,\ldots,S \qquad (34)$$

where T_{YL}^z denotes the rate of tax on labour income in region z. Similarly, for property income,

$$r_{GYP}^z = t_{YP}^z + y_P^z \qquad z = 1,\ldots,S \qquad (35)$$

where T_{YP}^z denotes the rate of tax on property income in region z.

The direct tax regime is characterized by the equations

$$t_{YL}^{z} = f_{Y}^{z} + f_{YL}^{z} \qquad z = 1,\ldots,S \qquad (36)$$

$$t_{YP}^{z} = f_{Y}^{z} + f_{YP}^{z} \qquad z = 1,\ldots,S \qquad (37)$$

where f_{Y}^{z} represents a general shift in income tax rates, f_{YL}^{z} a shift in the labour income tax rate, and f_{YP}^{z} a shift in the property income tax rate in region z.

Government net interest receipts are equal to the product of the bond rate and government net ownership of bonds:

$$R_{G}^{z} dQ_{I}^{RGz} = A_{B}^{Gz} dR_{B}^{z} + R_{B}^{z} R_{G}^{z} dQ_{B}^{YGz} \qquad z = 1,\ldots,S \qquad (38)$$

Government current outlays are equal to the sum of government consumption expenditure and transfer payments to households:

$$O_{G}^{z} o_{G}^{z} = Z_{G}^{Z} z_{G}^{z} + T_{G}^{z} t_{G}^{z} \qquad z = 1,\ldots,S \qquad (39)$$

where O_{G}^{z} denotes government current outlays. Transfer payments to households are equal to the product of a transfer payment shift variable and household pre-transfer income:

$$t_{G}^{z} = r_{2}^{z} + y_{V}^{Hz} \qquad z = 1,\ldots,S \qquad (40)$$

where r_{2}^{z} denotes the transfer payment shift variable and y_{V}^{Hz} denotes household pre-transfer income. Household pre-transfer income is equal to labour income plus household property income, less income tax:

$$Y_{V}^{Hz} y_{V}^{Hz} = Y_{L}^{z} y_{L}^{z} + Y_{P}^{Hz} y_{P}^{Hz} - R_{GY}^{z} r_{GY}^{z} \qquad z = 1,\ldots,S \qquad (41)$$

Government receipts are equal to the sum of government current outlays and government surplus on current transactions:

$$R_{G}^{z} r_{G}^{z} = O_{G}^{z} o_{G}^{z} + R_{G}^{z} dQ_{S}^{YGz} + S^{Gz} r_{G}^{z} \qquad z = 1,\ldots,S \qquad (42)$$

When all tax rates are exogenous, this equation implicitly determines government surplus on current transactions. When the government surplus ratio is exogenous and some tax rate variable (typically f_{Y}^{z}) is endogenous, the equation determines government revenue r_{G}^{z}, and through it the endogenous tax rate variable.

With the addition of these equations, some of the old equations for government receipts and outlays become redundant, and are discarded: (S35), income tax, (S37), aggregate government revenue, (S38), real transfers to households, and (S92), the ratio of net factor income to transfer payments to households. A small change is required to the labour supply equation, (S17); the old income tax rate variable is replaced by the new labour-specific income tax rate.

International allocation of investment

The treatment of the international allocation of investment is based on the same parity conditions as the international allocation of capital. The parity conditions state that capital is allocated in such a way that rates of return are equal between regions. To maintain this equality over time, investment must be allocated in such a way that time rates of change in rates of return are equal between regions.

For short-run applications, we recognize that rates of return in individual industries may deviate temporarily from parity. We assume that investment is concentrated in regions with abnormally high rates of return, away from regions with abnormally low rates, so that abnormal returns are gradually eliminated over time.

We begin by determining the expected rate of change in the rate of return on capital. We make several assumptions about investors' expectations. The expected rate of return on capital at any given point in time depends only on the size of the capital stock. Because the world economy grows over time, investors expect that capital stocks can also grow at some positive rate without a decline in the rate of return. The elasticity of the expected gross rate of return on capital with respect to the expected size of the capital stock is fixed. The rate of growth in the capital stock consistent with an expectation of no change over time in the rate of return is also fixed.

Under the stated assumptions, we can write for the following equation for the expected rate of return:

$$\frac{R_{KG}^{Ez}}{R_{KG}^{z}} = \left[\frac{F_{D2}^{Ez}}{\exp(-H^{z}t)F_{D2}^{z}}\right]^{-\alpha^{z}} \qquad z = 1,\ldots,S$$

where R_{KG}^{Ez} denotes the expected gross rate of return on capital in region z (averaged across industries), R_{KG}^{z} the actual average gross rate of return on capital, and F_{D2}^{Ez} the expected capital stock. The parameter α^{z} represents the absolute magnitude of the elasticity of the expected rate of return with respect to the expected capital stock, and H^{z} represents the rate of growth in the capital stock consistent with zero expected change over time in the rate of return. The limit of the expected gross rate of return as the expected size of the capital stock approaches infinity is zero; the limit as the expected capital stock approaches zero is infinity.

From this underlying equation, we derive an equation for the expected rate of change over time in the rate of return on capital:

$$d\dot{R}_K^{Ez} = -\alpha^z R_{KG}^z J^z (inv_{TR}^z - f_{D2}^z) - \alpha^z (J^z - R_D^z - H^z) dR_K^z \qquad z = 1,\ldots,S \qquad (43)$$

where $\dot{R}_K^{Ez}$ represents the expected rate of change in the rate of return on capital in region z, R_K^z the average gross rate of return on capital, J^z the ratio of gross investment to the size of the existing capital stock, and INV_{TR}^z gross investment. The first term on the right hand side shows that, the higher the level of investment, the more rapid the expected decline in the rate of return. The explanation for the second term is more complex. If the factor $(J^z - R_D^z - H^z)$ is not equal to zero, then the capital stock is not growing at the rate H^z consistent with zero expected change in the rate of return. Because of the constant-elasticity form of the underlying expected-rate-of-return equation, the expected rate of change in the rate of return is is proportional not only to the factor $(J^z - R_D^z - H^z)$, but also to the actual gross rate of return. The greater the actual rate of return, for any given rate of growth in the capital stock, the greater the expected rate of change in the rate of return. The derivation of equation (43) is given in Appendix B.

For equation (43), we need to determine the average rate of return on capital in each region:

$$dR_K^z = \sum_{j=1}^{J} S_{D2j}^z dR_{Kj}^z + \sum_{j=1}^{J} R_{Kj}^z S_{D2}^z f_{2j}^z - R_K^z f_{D2}^z \qquad z = 1,\ldots,S \qquad (44)$$

where S_{D2j}^z denotes the share of industry j in the capital stock of region z, and f_{2j}^z the capital stock in region z. The first term on the right hand side of equation (44) represents the contribution to change in the average rate of return of changes in rates of return in individual industries. The second and third terms represent the contribution of change in the composition of the capital stock, between higher return and lower return industries.

The next step is to specify the process by which rates of return equilibrate over time. We express the expected average rate of return on capital as the sum of the expected normal rate of return and the expected average abnormal rate:

$$R_K^{Ez} = R_E^{Ez} + R_A^{Ez}$$

where R_E^{Ez} denotes the expected normal rate of return (identified with the required rate of return on equity), and R_A^{Ez} the expected average abnormal rate. We assume that the average abnormal rate is expected to regress to zero over time by a Koyck process:

$$\dot{R}_A^{Ez} = -\lambda^z R_A^z$$

where $\dot{R}_A^{Ez}$ denotes the expected rate of change over time in the average abnormal rate of return, and λ^z represents a coefficient of adjustment. From these underlying equations, we obtain the equation for rate of return equilibration over time:

$$d\dot{R}_K^{Ez} = d\dot{R}_E^{Ez} - \lambda^z dR_A^z \qquad z = 1,\ldots,S \qquad (45)$$

where $\dot{R}_E^{Ez}$ denotes the expected rate of change in the required rate of return on equity in region z.

We determine the average rate of abnormal return on capital in each region by the equation:

$$dR_A^z = \sum_{j=1}^{J} S_{D2j}^z dR_{Aj}^z + \sum_{j=1}^{J} R_{Aj}^z S_{D2}^z f_{2j}^z - R_A^z f_{D2}^z \qquad z = 1,\ldots,S \qquad (46)$$

The expected rate of change in the required rate of return on equity is determined by an equation derived from the equity-bond rate parity condition:

$$d\dot{R}_E^{Ez} = d\dot{R}_B^{Ez} - d\dot{F}_{RE}^z \qquad z = 1,\ldots,S \qquad (47)$$

where $\dot{R}_B^{Ez}$ denotes the expected rate of change over time in the bond rate in region z, and $\dot{F}_{RE}^z$ the expected rate of change over time in the equity premium.

Finally, expected rates of change over time in bond rates in different regions are governed by the international interest parity condition, so

$$d\dot{R}_B^{Ez} = d\dot{R}_B^E \qquad z = 1,\ldots,S \qquad (48)$$

where $\dot{R}_B^E$ denotes the expected rate of change over time in the world bond rate.

Equations (43), (45), (47) and (48) together define a global investment schedule, determining global investment as a decreasing function of the expected rate of change in the world bond rate. The lower the expected future bond rate, the higher the level of investment. The investment schedule also defines an inverse demand function for investment, showing the expected rate of change in the world bond rate consistent with a given level of investment.

Within the model, global investment is determined by global saving. Through the inverse demand function, global investment determines the expected rate of change in the world bond rate. This in turn determines investment in each region.

4 Modifications to the SALTER database

Additional data required for the the new model include net interest payments by governments and net national property income from abroad. Income taxation is separated into taxation of labour income and property income.

In the new model, rates of tax on property income can be important determinants of model results. This is because the net benefits to each region of foreign capital inflow depend critically on the property tax rate in the region. The higher the property tax rate, the greater the net benefit from capital inflow.

In preparing the database for the old model, where income tax rates were less important, income tax was treated as a slack component in the government budget. For the new model, this is not viable.

To allow asset values to be calculated from property income flows, the new database includes an element representing the world-wide pre-tax interest rate. For use in short-run simulations, the database contains data for abnormal earnings of capital, by industry and region. The full set of additional database arrays is shown in Table 1.

For the first implementation of the capital mobility extension, these arrays are constructed as follows.

Entries for government net interest receipts (MK03), tax on labour income (MK04), tax on non-labour income (MK05), and the world real bond rate (MK06) are based directly on relevant economic statistics.

Table 1: **New database arrays for the capital mobility extension**

Header	*Dimensions*	*Description*	*Units*
MK01	*J* x *S*	Abnormal capital returns	1988 US$ million
MK02	*S*	Household net interest income	1988 US$ million
MK03	*S*	Government net interest receipts	1988 US$ million
MK04	*S*	Tax on labour income	1988 US$ million
MK05	*S*	Tax on non-labour income	1988 US$ million
MK06	1	World real bond rate	per year
MK07	*S*	Real rate of return on equity	per year
MK08	*S*	Exchange rate	Multiple of base-case exchange rate
MK09	*S*	Rate of growth in real household income	per year
MK10	*S*	Length of simulation period	years
MK11	*S*	Rate of growth in real government receipts	per year
MK12	*S*	Rate of return adjustment coefficient	per year
MK13	*S*	Elasticity of expected gross rate of return	(none)
MK14	*S*	Steady state capital stock growth rate	per year

J number of industries
S number of regions

Entries for household net interest income (MK02) are constructed in accordance with the assumption in the theoretical structure of the model that all international property income flows are interest income. Then

net property income from abroad

= net interest income from abroad

= net national interest income

(since interest payments between domestic residents cancel out)

= (household net interest income) + (government net interest receipts)

so

household net interest income

= (net property income from abroad) - (government net interest receipts)

This formula is used to calculate household net interest income, using economic statistics for net property income from abroad and government net interest receipts.

To reflect initial long-run equilibrium, entries for abnormal capital returns (MK01) are all set at zero. In accordance with the absence of risk and risk aversion from the theoretical structure, entries for real rates of return on equity (MK07) are set equal to the world real bond rate (MK06).

It would be possible to set rates of return on equity above the bond rate, in order to reflect a realistic equity premium. This is not advised, since with risk not accounted for in the theoretical structure, non-zero equity premiums would be liable to generate free lunches. Specifically, a country with a positive equity premium would get a free lunch from debt-financed domestic equity investment. Returns from the equity investment would exceed the interest expense by an amount corresponding to the equity premium. But the model would ignore the consideration which gives rise to equity premiums in reality, namely, that equity investment is subject to risks which reduce its certainty equivalent rate of return to a level matching investment in bonds. Accounting for the benefit of the equity premium but not the offsetting cost of the risk would lead to spurious welfare effects.

Entries for exchange rates (MK08) are set equal to unity. This reflects the fact that all input-output data in the initial database are expressed in common units, 1988 US dollars. The theoretical structure, however, does provide for exchange rate changes. This makes it necessary to include exchange rates in the database, so that the model can handle differences in units of account across regions.

Entries for rates of growth in real household income (MK09) and real government receipts (MK11), expressed as a fraction rather than a percentage, are initially based on statistics for rates of growth in real gdp in each region. To obtain typical rather than historical-year growth rates, the entries are based as far as possible on average rates of growth over the ten-year period 1978-88.

The appropriate setting of the length of simulation period (MK10) depends on the application. For long-run simulations, a typical setting might be 10.0 (representing ten years). For short-run simulations, users should set the simulation period at zero.

For the initial implementation, the rate of return adjustment coefficients (MK12) have been set at 0.2. This parameter is relevant to simulations in which abnormal returns on capital are non-zero. These are typically short-run simulations The international allocation of capital in these simulations is controlled by a partial adjustment process, under which abnormal returns are gradually eliminated over time. The setting of 0.2 means that investment in each region occurs at a rate such that 20 per cent of the abnormal component of the rate of return is expected to be eliminated within a year.

The partial adjustment equations also involve parameters representing the elasticity of the expected gross rate of return on capital in each region with respect to the size of the capital stock (MK13). Thus they represent elasticities perceived by investors. The parameters are set so that the perceived elasticities agree with actual elasticities implicit in the initial model.

The implicit actual elasticities are obtained from simulations with a trial database. Shocks of 1.0 are applied to the equity premium shift variables (*frer*) country by country. The elasticities can then be calculated as

$$1.0/\{[RTEQR(z) + RTDPR(z)].kt(z)\}$$

where *RTEQR*(*z*) represents the rate of return on equity, *RTDPR*(*z*) the depreciation rate, and *kt*(*z*) the percentage change in the capital stock in region *z*. These elasticities are then incorporated in the database, and the simulation repeated with the revised datbase. Iteration continues until satisfactory convergence is obtained.

The entries under header MK14, the 'steady-state capital stock growth rate', are used in the theoretical structure to represent that rate of growth in the capital stock in each region which would be consistent with zero change over time in the rate of return on capital. In a database calibrated to represent a long-run equilibrium, the actual rates of growth implicit in the database would be close to these steady-state growth rates. But the databse is not in fact calibrated. To prevent deviations from long-run equilibrium from colouring investment results, the entries for array MK14 are constructed to be equal to the actual rates of growth in capital stocks implicit in the database.

Appendix A: The wealth accumulation equations

In this appendix we derive the wealth accumulation equations (13) and (15) presented in Section 3.

We derive the wealth accumulation relation by integrating the time rate of change in real wealth through the simulation period. To do this we need information about the time paths of various economic variables through the period. Since the model determines only the end-of-period values for these variables, we need to make various assumptions about the time paths leading up to these end-of-period values.

Assumptions

In stating the assumptions we use *constant* to mean constant over time, *exogenous* to mean fixed across alternative states of the economy at a given point in time.

We assume:

(a1) all prices grow over the simulation period at a common exogenous rate,

(a2) asset quantities at the beginning of the simulation period are exogenous,

(a3) real income at the beginning of the simulation period is exogenous,

(a4) real income grows over the simulation period at a constant endogenous rate, and

(a5) the saving-income ratio is constant over the simulation period.

These assumptions are motivated partly by tractability and partly by realism. The assumption (a1) that all prices grow over the simulation period at a common exogenous rate makes the problem more tractable, by ruling out real capital gains over the simulation period. This allows us to avoid specifying separate time paths for different classes of assets that might generate different rates of real capital gain.

At the same time the assumption has some merit with respect to realism. It implies that external shocks have a one-off effect on prices at the beginning of the simulation period, but do not affect growth in prices through the period. Given the tendency of prices to overshoot in response to economic shocks over the short run, this appears generally more realistic than the alternative assumption, that external shocks feed gradually into prices through the simulation period.

The assumptions (a3 and a4) concerning real income imply that external shocks have no effect on real income initially, but feed gradually into real income through the simulation period. Given that the initial impact of external shocks on aggregate real activity is often opposite in direction to the longer-run impact, this appears more realistic than the alternative, of assuming that the impact at the beginning of the simulation period is similar to the impact at the end of the period.

Notation

We let T denote the length of the simulation period. We measure time from the beginning of the simulation period, so the period covers times $t \in [0, T]$. We denote the time to which a variable refers by a subscript; thus X_t denotes the value of variable X at time t after the beginning of the simulation period.

A dot placed above a variable indicates the derivative of the variable with respect to time. A circumflex placed above a variable indicates the rate of growth in the variable over time. Thus for any variable X,

$$\hat{X} = \frac{\dot{X}}{X}$$

In general, the conversion of a symbol from upper-case to lower-case indicates the relative change in the original variable across alternative states of the economy. Thus

$$x = \frac{dX}{X}$$

An exception to this rule applies to variables representing time, where the upper-case variable T denotes the length of the simulation period, and the lower-case variable t any time within the adjustment period.

Derivation

Let A denote real wealth, and S real saving. The rate of change over time in real wealth is equal to real saving:

$$\dot{A}_t = S_t \tag{A1}$$

Let Y denote real income, and Z the saving ratio. By the assumption (a5) that the saving ratio is constant over the simulation period, we have at each time t,

$$S_t = ZY_t \tag{A2}$$

By the assumption (a4) that the rate of growth in real income is constant over the simulation period, we have

$$Y_t = Y_0 e^{\hat{Y} t} \tag{A3}$$

Combining equations (A1)-(A3), we have

$$\dot{A}_t = Z Y_0 e^{\hat{Y} t} \tag{A4}$$

Integrating equation (A4) over the simulation period, and assuming that the rate of growth in real income $\hat{Y}$ is not equal to zero, we obtain

$$A_T = A_0 + \frac{Z Y_0}{\hat{Y}} (e^{\hat{Y} T} - 1) \tag{A5}$$

Totally differentiating equation (A5), and assuming that $T \neq 0$, we obtain

$$\begin{aligned} A_T a_T = {} & A_0 a_0 + \frac{e^{\hat{Y} T} - 1}{\hat{Y} T} . Y_0 T . dZ + \frac{e^{\hat{Y} T} - 1}{\hat{Y} T} . Z Y_0 T . y_0 \\ & + \frac{\hat{Y} T e^{\hat{Y} T} - e^{\hat{Y} T} + 1}{(\hat{Y} T)^2} . Z Y_0 T . T d\hat{Y} \end{aligned} \tag{A6}$$

Equation (A6) represents the wealth accumulation relation that we seek. Before incorporating it in the model, however, we need to make various substitutions in it.

In the first set of substitutions we replace the beginning-of-period values A_0 and Y_0 with expressions involving end-of-period values. We also replace the through-the-period variable $d\hat{Y}$ with an expression involving beginning- and end-of-period variables.

Putting $t = T$ in equation (A3), totally differentiating and rearranging, we obtain

$$d\hat{Y} = \frac{y_T - y_0}{T} \tag{A7}$$

Also putting $t = T$ in equation (A3) and inverting, we obtain

$$Y_0 = e^{-\hat{Y} T} Y_T \tag{A8}$$

Lastly from equation (A5) we have

$$A_0 = A_T - \frac{e^{\hat{Y} T} - 1}{\hat{Y} T} Z . Y_0 T \tag{A9}$$

Substituting from equation (A8) into (A9) and rearranging, we obtain

$$A_0 = A_T - \frac{1-e^{-\hat{Y}T}}{\hat{Y}T} Z.Y_T T \tag{A10}$$

Substituting from equations (A7), (A8) and (A10) into (A6) and rearranging, we obtain

$$\begin{aligned} A_T a_T = {} & \left(A_T - \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.ZY_T T \right) a_0 + \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.Y_T T.dZ + \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.ZY_T T.y_0 \\ & + \frac{\hat{Y}T - 1 + e^{-\hat{Y}T}}{(\hat{Y}T)^2}.ZY_T T.(y_T - y_0) \end{aligned} \tag{A11}$$

In the next set of substitutions we replace the real variables a and y with expressions involving the corresponding nominal variables and prices. Let N denote nominal wealth and P a price deflator used to convert income and wealth from nominal to real terms. Because of assumption (a1) we may write

$$p_T = p_0 = p$$

We may express real assets at any time t as the quotient of nominal assets and the price deflator,

$$A_t = \frac{N_t}{P_t} \tag{A12}$$

Totally differentiating equation (A12), we obtain

$$a_t = n_t - p \tag{A13}$$

Likewise, using I_t to denote nominal income at time t, we have

$$Y_t = \frac{I_t}{P_t} \tag{A14}$$

and

$$y_t = i_t - p \tag{A15}$$

Substituting from (A13) and (A15) into (A11), cancelling terms, and multiplying through by the end-of-period price deflator P_T, we obtain

$$N_T n_T = \left(N_T - \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.ZI_T T \right) n_0 + \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.I_T T.dZ + \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.ZI_T T.i_0$$
$$+ \frac{\hat{Y}T - 1 + e^{-\hat{Y}T}}{(\hat{Y}T)^2}.ZI_T T.(i_T - i_0) \qquad \text{(A16)}$$

In the final set of substitutions we use the assumptions about the time path of adjustment to eliminate beginning-of-period variables. We also substitute for the saving ratio Z with an expression involving end-of-period saving.

Let F denote an asset price index. By the assumption (a2) that the initial quantity of assets is exogenous, we have

$$n_0 = f \qquad \text{(A17)}$$

Also by the assumption (a3) that initial real income is exogenous, we have

$$y_0 = 0 \qquad \text{(A18)}$$

Substituting from equation (A18) into (A15) for $t = 0$ and rearranging, we obtain

$$i_0 = p \qquad \text{(A19)}$$

Lastly putting $t = T$ in equation (A2), we have

$$Z = \frac{S_T}{Y_T} \qquad \text{(A20)}$$

Let M denote the money value of saving. Then we have

$$S_t = \frac{M_t}{P_t} \qquad \text{(A21)}$$

Substituting from equations (A14) and (A21) into (A20) and cancelling factors, we obtain

$$Z = \frac{M_T}{I_T} \qquad \text{(A22)}$$

Substituting from equations (A17), (A19) and (A22) into (A16), we obtain

$$N_T n_T = \left(N_T - \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.M_T T\right)f + \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.I_T T.dZ$$
$$+ \frac{1-e^{-\hat{Y}T}}{\hat{Y}T}.M_T T.p + \frac{\hat{Y}T - 1 + e^{-\hat{Y}T}}{(\hat{Y}T)^2}.M_T T.(i_T - p) \qquad \text{(A23)}$$

We define coefficients

$$C_1 = \frac{1-e^{-\hat{Y}T}}{\hat{Y}T} \qquad \text{(A24)}$$

$$C_2 = \frac{\hat{Y}T - 1 + e^{-\hat{Y}T}}{(\hat{Y}T)^2} \qquad \text{(A25)}$$

Using these we can rewrite equation (A23) as

$$N_T n_T = (N_T - C_1.M_T T)f + C_1.I_T T.dZ + C_1.M_T T.p$$
$$+ C_2.M_T T.(i_T - p) \qquad \text{(A26)}$$

This is the normal final form of the wealth accumulation relation. We still have, however, to cover the special cases $\hat{Y}=0$ and $T=0$. To cover these, we define, when $\hat{Y}T=0$,

$$C_1 = 1 \qquad \text{(A24')}$$

$$C_2 = \tfrac{1}{2} \qquad \text{(A25')}$$

These are the limits of C_1 and C_2 as $\hat{Y}T$ approaches zero, derivable using l'Hopital's rule. Then when $\hat{Y}=0$, equation (A26) reduces to

$$N_T n_T = (N_T - M_T T)f + I_T T.dZ + M_T T.p + \tfrac{1}{2}.M_T T.(i_T - p) \qquad \text{(A26')}$$

By applying to the case $\hat{Y}=0$ the procedure followed above to derive equation (A26) for the case $\hat{Y}\neq 0$, the reader can verify that equation (A26′) is valid.

When $T=0$, equation (A26) reduces to

$$n_T = f \qquad \text{(A26'')}$$

as is clearly required.

In implementing the equation system we presume that net household assets will always be positive. We allow, however, for the possibility that net government assets may be positive, negative, or zero. Accordingly we avoid the percentage change formulation for the government wealth variable.

Putting Q for the ratio of wealth to income, we have

$$N_t = Q_t I_t \tag{A27}$$

Putting $t = T$ in equation (A27) and totally differentiating, we obtain

$$N_t n_t = I_T dQ_T + N_T i_T \tag{A28}$$

So we may rewrite equation (A26) as

$$\begin{aligned} I_T dQ_T + N_T i_T = {} & (N_T - C_1 . M_T T) f + C_1 . I_T T . dZ + C_1 . M_T T . p \\ & + C_2 . M_T T . (i_T - p) \end{aligned} \tag{A29}$$

We use this as the basis for the government wealth accumulation equation (15) in the model.

Implementation

To incorporate the wealth accumulation relation in the equation system we need to identify the coefficients and variables appearing in it with coefficients and variables appearing in the model. We need to make two sets of identifications, one for the household sector and one for government.

For the coefficients the identifications are obvious. The variables, however, require some consideration.

For the household nominal income variable we use household disposable income. For the government nominal income variable we use government revenue (net of indirect subsidy payments and interest expense). For the price deflators used to convert nominal to real income we use the household and government consumption price indices.

We also need to specify variables in the model corresponding to asset price indices. For capital and land we have asset prices already available in the model. Under the assumption that equity prices reflect the prices of underlying physical assets, we can construct the price of equity as an index of capital and land prices. The construction of bond prices, however, requires more consideration.

It might seem natural to define the net quantity of bonds held in each region as the value of the holdings in the local currency. This would imply that the price of bonds would always be identically unity. The failing of this approach is that it would create inconsistencies in the quantity of bondholdings between regions. Since one country's lendings are anothers' borrowings, we need a common quantity unit for the net bondholdings of all regions.

We can construct such a unit by assuming that all bonds are denominated in the neutral currency. We do this, and further assume that all bonds are indexed to the world consumption price index. In so doing we enable the capital mobility extension to preserve the price homogeneity properties of the model: an increase in the world price level, or an increase in the domestic price level accompanied by a currency depreciation, confers no benefit on debtor regions, and imposes no cost on creditor regions.

Of course price homogeneity is not necessarily a merit in a model with financial assets; unanticipated price changes do in reality redistribute wealth between debtors and creditors. But in this model it has several benefits. The price homogeneity property is a useful check against errors in implementing the theoretical structure, which we wish to preserve. It also makes model results easier to understand, by excluding a potential new source of welfare gains and losses in individual regions. Finally, we recall that the model lacks a monetary sector, and that the world price level and the price level in each country are set arbitrarily through the closure. Furthermore, the theoretical structure and database do not specify the currencies in which international borrowing is denominated. It is better then to abstract from the redistributive effects of price level changes than to introduce effects which would be driven by arbitrary assumptions about price levels and bond denominations.

Following this approach, we define the price of bonds in each region as the world consumption price index converted to local currency units. For government, which holds only bonds, the asset price index is just the bond price. For households, which hold both equity and bonds, the asset price index is an index of equity and bond prices.

Implementation of the wealth accumulation relations also requires the addition to the database of two new parameters for each region. These are the coefficients corresponding to the coefficient $\hat{Y}$ in this appendix, namely the base-case rates of growth in household disposable income and government revenue.

Appendix B: Expected rate of change in the rate of return on capital

This appendix derives the equation in the theoretical structure for the expected rate change in the rate of return on physical capital, equation (43) of Section 3.

Usage and notation

Expectations are those held by investors at the terminal instant. The *future* consists of times later than the terminal instant.

A dot placed above the symbol for a variable indicates differentation of the variable with respect to time. Thus for a variable X, $\dot{X}$ denotes the time rate of change in X.

In general, the conversion of a symbol from upper-case to lower-case indicates the relative change in the original variable across alternative states of the economy. Thus

$$x = \frac{dX}{X}$$

Assumptions

(a1) The elasticity of the expected gross rate of return on capital with respect to the size of the capital stock is exogenous, constant, and negative.

(a2) The rate of growth in the capital stock that in investors' expectations would be consistent with a constant rate of return is exogenous.

(a3) Investors are accurately informed of current circumstances, including the current size of the capital stock, rate of return on capital, and level of investment.

(a4) The depreciation rate is exogenous.

Derivation

Let R_G^E denote the variable representing the expected gross rate of return on capital, and $\overline{R}_G$ the actual gross rate of return at the terminal instant. Let K^E denote the expected capital stock, and K_C the future capital stock which in investors' expectations would be consistent with the future gross rate of return being equal to $\overline{R}_G$. Let α denote the absolute magnitude of the elasticity of the expected future rate of return with respect to the future capital stock. By assumption (a1) we may treat this as a fixed coefficient. Then we have

$$\frac{R_G^E}{\overline{R}_G} = \left(\frac{K^E}{K_C}\right)^{-\alpha} \tag{B1}$$

Let $\overline{K}$ denote the actual size of the capital stock at the terminal instant. Let κ denote the rate of growth in the capital stock which in investors' expectations would be consistent with the rate of return being constant over time. By assumption (a2) we may treat this as a fixed coefficient. Then we have

$$K_C = e^{\kappa t}\overline{K} \tag{B2}$$

Substituting from equation (B2) into (B1), we obtain

$$\frac{R_G^E}{\overline{R}_G} = \left(\frac{K^E}{e^{\kappa t}\overline{K}}\right)^{-\alpha} \tag{B3}$$

Differentiating with respect to time, dividing through by equation (B3), putting R_G for the base case value of R_G^E at the terminal instant, and rearranging, we obtain

$$\dot{R}_G^E = -\alpha R_G^E\left(\frac{\dot{K}^E}{K^E} - \kappa\right) \tag{B4}$$

Let δ denote the depreciation rate, and R^E the expected net rate of return. Then we have

$$R_G^E = R^E + \delta \tag{B5}$$

Differentiating with respect to time, and appealing to the assumption (a4) that the depreciation rate is exogenous, we obtain

$$\dot{R}_G^E = \dot{R}^E \tag{B6}$$

Also let R_G denote the actual gross rate of return at the terminal instant. We may assume (a3) that the rate of return expected to apply at the terminal instant is equal to the actual rate at the terminal instant. Then we have at the terminal instant

$$R_G^E = R_G \tag{B7}$$

Similarly we have at the terminal instant

$$K^E = K \qquad \text{(B8)}$$

and

$$\dot{K}^E = \dot{K} \qquad \text{(B9)}$$

Substituting from equations (B6)-(B9) into equation (B4), we obtain for the terminal instant

$$\dot{R}^E = -\alpha R_G \left(\frac{\dot{K}}{K} - \kappa \right) \qquad \text{(B10)}$$

Now let I denote investment. Then we have

$$\dot{K} = I - \delta K \qquad \text{(B11)}$$

Let J denote the ratio of investment to the capital stock. Then we have

$$J = \frac{I}{K} \qquad \text{(B12)}$$

Solving for I and substituting into equation (B11), we obtain

$$\dot{K} = (J - \delta)K \qquad \text{(B13)}$$

Then substituting into (B10), we obtain

$$\dot{R}^E = -\alpha R_G (J - \delta - \kappa) \qquad \text{(B14)}$$

This is the final levels form of the equation for the expected rate of change in the rate of return on capital.

Totally differentiating across alternative states of the economy, we obtain

$$d\dot{R}^E = -\alpha R_G J j - \alpha (J - \delta - \kappa) dR_G \qquad \text{(B15)}$$

Now we have for the actual gross rate of return,

$$R_G = R + \delta \qquad \text{(B16)}$$

Totally differentiating, and applying the assumption (a4) that the depreciation rate is exogenous, we obtain

$$dR_G = dR \qquad \text{(B17)}$$

Also totally differentiating equation (B12), we obtain

$$j = i - k \qquad \text{(B18)}$$

Substituting from equations (B17) and (B18) into equation (B15), we obtain

$$d\dot{R}^E = -\alpha R_G J(i - k) - \alpha(J - \delta - \kappa)dR \tag{B19}$$

Rewritten in the notation used for the theoretical structure, this becomes equation (43) of Section 3.

Implementation

In implementing this equation in the model, some consideration needs to be given to parameter settings. The new parameters needed for the equation are the elasticity of the expected rate of return α, and the constant-expected-rate-of-return rate of growth in the capital stock, κ.

For the initial implemention of the model, we assume that the constant-expected-rate-of-return rate of growth in the capital stock is equal to the actual rate of growth in the capital stock implicit in the database,

$$\kappa = j - \delta$$

Then for small-change simulations, equation (B19) reduces to

$$d\dot{R} = -\alpha R_G J(i - k)$$

We assume that the elasticity of the expected rate of return, α, is equal to the actual elasticity of the rate of return. To express this in terms of model variables and pre-existing elements of the database, we totally differentiate equation (B3), divide the resultant equation by (B3), and rearrange, obtaining

$$\alpha = -\frac{1}{R_G} \cdot \frac{dR}{k}$$

Here dR and k correspond to variables in the model, while the gross rate of return R_G can be calculated from the database. We simulate a change in the required rate of return on capital brought about through a shift in the equity premium (variable dF^z_{RE} in the theoretical structure) observe dR and k, calculate α, insert it into the database, repeat the simulation, and iterating until α converges satisfactorily (convergence is typically very rapid).

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