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A SALTER Database Aggregation Facility

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A SALTER DATABASE AGGREGATION FACILITY

This paper describes a SALTER database aggregation facility.

SALTER is a multi-region multi-sector model of the world economy first documented in Jomini *et al.* (1991). It is designed as an instrument for policy analysis, in particular, for analysing commercial and industry policy issues in an international context.

The SALTER database incorporates a multi-region input-output table, some additional national accounts data, and a collection of behavioural parameters. At the time of writing, the database distinguishes sixteen regions, and thirty-seven industries in each region. This makes SALTER one of the more detailed multi-country general equilibrium models currently in use.

While the relatively fine disaggregation of the database is one of the strengths of the model, there are times at which it is inconvenient. The size of the database adds to the model's solution time, and makes simulation results harder to analyse. For issues for which detailed representation of regions, industries and commodities is not critical, the essential insights from the model could be gained more quickly and easily using a smaller database.

The SALTER database aggregation facility allows model users to reduce the size of the database as needed to suit their particular application. It performs user-specified aggregations of the regional classification, the sectoral classification, or both. The aggregation covers both the multi-region input-output data and the behavioural parameters of the original model. It does not cover the parameters required to support the capital mobility extension (McDougall 1993) although the user can easily add their own aggregated capital mobility parameters to the database produced by this aggregation facility.

For the behavioural parameters, the facility supports two styles of aggregation. One style is designed to minimise divergences in simulation results between the aggregated database and the original unaggregated database. The other style is designed to preserve in the aggregated database the parameter setting principles applied in constructing the standard database.

This paper describes the specifications and use of the aggregation facility. Section 1 provides background information about the SALTER model. Section 2 outlines the principles followed in designing the facility. Section 3 provides detailed specifications for the facility, and Section 4 describes the implementation and use of the facility.

1 SALTER background

The SALTER theoretical structure imposes various restrictions on demand systems within the model which must be taken into account in designing the aggregation procedure. These include both restrictions on functional forms for individual demand systems, and restrictions on parameter values imposed across demand systems.

In SALTER, each commodity can be used for many different purposes in each region. The different uses of each commodity include intermediate usage by each industry, household consumption, government consumption, and investment. Each use category in each region has its own separate demand system.

Within SALTER we specify several source-specific varieties of each commodity. Each region produces a distinct variety of each commodity. For each use category, there is a nested demand system. At the top level, demand is allocated across commodities; at the next level, between the domestic variety and imports of each commodity. For imports there is one further level, in which demand is allocated between imports from different regions.

SALTER imposes various functional forms on all these demand systems at all levels. The functional form imposed at the top level is fixed coefficients (Leontief) for most demand categories, except for household consumption, for which it is the linear expenditure system (LES). For domestic-import substitution it is constant elasticity of substitution (CES); for import-import substitution it is also CES.

The model also contains demand systems allocating demand for primary factors by each industry between labour, capital, and land. Once again these systems are CES.

Besides restricting the functional form of the various demand systems, the theoretical structure also imposes cross-system restrictions on parameter values. Specifically, it requires substitution elasticities in intermediate usage to be uniform across industries. The theoretical structure provides for a single substitution elasticity for each commodity and region to be read from the database for all industries; so that elasticities of substitution in intermediate usage can vary across commodities and regions, but not across industries within each region.

2 Overview of the aggregation facility

Aggregating the input-output flows is straightforward. Aggregating the behavioural parameters involves theoretical difficulties and trade-offs between different objectives. To give users some freedom to pursue their own objectives, the facility supports two different styles of aggregation, differing in their treatment of the behavioural parameters.

With the first style, the objective is fidelity in simulation results. Ideally, simulation results obtained with the aggregated database would be fully consistent with results with the original database. That is, aggregating the database and then running the simulation should give the same results as running the simulation and then aggregating the results. In general, this ideal cannot be attained, so the objective is to approach it as closely as possible. We call this style *outcomes-oriented aggregation.*

With the second style of aggregation, the objective is consistency with the original parameter-setting procedure. Most of the parameter values in the standard SALTER database were obtained by searching the empirical literature for parameter estimates, and then mapping from the regional and sectoral classifications used in the literature to the SALTER classifications. With the second style of aggregation, we aim to replicate in the aggregated database the parameter values that would have been obtained if the original parameter-setting procedure had been applied to the aggregated sectoral classification. We call this style *process-oriented aggregation.*

With both styles of aggregation, we preserve all functional forms specified in the standard model. The consumer demand systems in the aggregated model are based on the linear expenditure system, as in the standard model.

Similarly, where constant elasticity of substitution (CES) aggregator functions are used in the standard model, the same functional form is applied in the aggregated model. The need to preserve functional forms raises no difficulty in process-oriented aggregation, but does in outcomes-oriented aggregation. These and other difficulties are discussed further below.

Process-oriented aggregation

With process-oriented aggregation, we calculate behavioural parameters for the aggregated database as simple share-weighted averages of parameters from the original database.

Consider first the domestic-import substitution elasticities. In the original database, let σ^O_{kzi} denote the elasticity of substitution between imported and domestic varieties of commodity i in use k (k ranging over intermediate usage, investment, household consumption, and government consumption) in region z. Let σ^A_{kZI} denote the elasticity of substitution for aggregate commodity I in aggregate region Z. Then we set

$$\sigma^A_{kZI} = \sum_{z \in Z} \sum_{i \in I} S_{kzi} \sigma^O_{kzi}$$

where S_{kzi} denotes the share of expenditure on commodity i in region z in total expenditure on commodity I in region Z for purpose k.

Note that here and throughout the aggregation facility, in calculating expenditure shares, we measure expenditure at purchasers' prices rather than the alternative, basic values.

We use the same shares in calculating the aggregate import-import substitution elasticities as weighted averages of the original import-import elasticities. In doing so, we reject one natural-seeming approach, of using shares in expenditure on imports, rather than shares in expenditure on domestic products and imports together. Using the same shares for the import-import elasticities as for the domestic-import elasticities preserves ratios between import-import and domestic-import elasticities. In process-oriented aggregation, we wish to preserve these ratios. This is because the import-import elasticities in the standard SALTER database are set by applying these ratios to the domestic-import elasticities.

For the factor substitution elasticities, we use as weights the share of each original industry and region in total factor earnings in each aggregated industry in each aggregated region. For the wage indexation parameters, we use shares in total labour earnings.

The form imposed on the uppermost level of the consumer demand system in the SALTER theoretical structure is the linear expenditure system. In this system, we have to decide which parameters to aggregate. We chose to aggregate the expenditure elasticities and the Frisch parameter. This is despite the fact that the expenditure elasticities are not fundamental to the consumer demand system, but are derived from the marginal budget shares, which are basic. The reason is that the marginal budget shares in the standard SALTER database were calculated from previously selected expenditure elasticities.

To aggregate the consumer demand parameters we therefore proceed as follows. First, we calculate the expenditure elasticities in the original database from the original marginal and average budget shares. Next, we calculate aggregate expenditure elasticities, using as weights the share of expenditure on each original commodity in each original region, in household consumption expenditure on the corresponding aggregate commodity in the corresponding aggregate region. We also calculate aggregate Frisch parameters, using as weights the share of each original region in household consumption expenditure in the corresponding aggregate region. Finally, from the aggregate expenditure elasticities and average budget shares we calculate aggregate marginal budget shares.

Outcomes-oriented aggregation

The objective in the outcomes-oriented style of aggregation is consistency in simulation results between the aggregated and the original model. Several theoretical problems arise in pursuing this objective. The sources of these problems are information losses in aggregation, restrictions on functional forms, and cross-system restrictions on parameter values.

Simulations with the aggregated model obviously use less information than simulations with the original database. Information losses are suffered in the scenario for predetermined variables, the contents of the database, and results for intermediate endogenous variables. Because of these losses, the aggregated model can only approximate the behaviour of the disaggregate model.

Even where information losses do not arise, problems may arise from restrictions on functional forms. As discussed in Section 1, the SALTER theoretical structure imposes restrictions on functional forms in many parts of the model. Unfortunately, aggregation does not generally preserve functional forms. Since the SALTER theoretical structure forces us to preserve functional forms in aggregation, we can only approximate the true aggregated functions.

Finally, problems arise from cross-system constraints built into the model structure. These constraints apply when many distinct systems in the model are parameterised with a single parameter value in the database. But these uniformity properties of the original theoretical structure are not always preserved under aggregation. The single parameter value to be supplied in the aggregated database must then be chosen as a compromise between the different values indicated for the theoretically distinct parameters.

The remainder of this section provides simple examples of each of these problems, and describes how they are dealt with in designing the aggregation facility.

Aggregation errors arising from information losses

The most obvious reason for aggregation error is the loss of information between the original and the aggregated model. We give two simple examples showing how such losses arise.

The first example shows how errors in aggregation can arise when behavioural parameters vary across sectors. The behavioural parameters we choose for the example are demand elasticities. For simplicity, we base the example not on complete demand systems such as all demand systems in SALTER, but on a single-price demand equation such as the export demand equation in ORANI (Dixon *et al.* 1982).

Consider a group of commodities $i \in I$. For each commodity i let η_i denote the own-price elasticity of demand. For a particular simulation, let x_i denote the percentage change in demand for commodity i (relative to the base case), and p_i the percentage change in price. Then, ignoring cross-price elasticities, we have

$$x_i = \eta_i p_i \qquad \text{(E1.1)}$$

In the aggregate model, we relate the percentage change in demand for the aggregate commodity *I*, x, to the percentage change in the price of the aggregate commodity, p, through an aggregate demand elasticity η. In interpreting results from the aggregate model, we assume that the aggregate variables represent simple weighted averages of the disaggregate variables:

$$x = \sum_i T_i x_i \qquad \text{(E1.2)}$$

$$p = \sum_i T_i p_i \qquad \text{(E1.3)}$$

where T_i denotes the share of commodity i in total expenditure on aggregate commodity *I* (in the base case).

Now for consistency in simulation results between the original and the aggregated model, we require equations (E1.1)-(E1.3) to be satisfied simultaneously. Substituting for x_i from (E1.1) into (E1.2), we obtain

$$x = \sum_i T_i \eta_i p_i$$

We may rewrite this as

$$x = \sum_i T_i \eta_i p + \sum_i T_i \eta_i (p_i - p)$$

Putting

$$\eta = \sum_i T_i \eta_i$$

we obtain

$$x = \eta p + \sum_i T_i(\eta_i - \eta)(p_i - p) \quad \left(\text{since } \sum_i T_i \eta (p_i - p) = 0\right)$$

$$= \eta p + Cov_{Ti}[\eta_i, p_i] \qquad \text{(E1.4)}$$

where $Cov_{Ti}[\eta_i, p_i]$ denotes the covariance of the demand elasticity η_i and the percentage change in price, p_i, calculated using expenditure weights.

Examining equation (E1.4.), we see that if the demand elasticity η_i or the percentage change in price, p_i, are uniform across commodities in the group, then $Cov_{Ti}[\eta_i, p_i]$ is equal to zero, and the percentage change x in demand for the aggregate commodity is given simply by

$$x = \eta p \tag{E1.5}$$

an equation of the same form as that applying to the original commodities, and involving only variables and parameters from the aggregated model.

On the other hand, when demand elasticities and percentage changes in prices vary across commodities in the group, the covariance term is typically non-zero. If the commodities with the largest price changes are those for which demand is most price elastic, then the effect of the price changes on quantity demanded is greater than indicated by equation (E1.5); if they are those for which demand is least price elastic, the effect on quantity demanded is smaller.

Now in solving the aggregated model, we obviously lack information on variation in prices across commodities contained within a single aggregated commodity. Accordingly, we cannot determine the covariance term $Cov_{Ti}[\eta_i, p_i]$ in equation (E1.4). Lacking this information, we make the neutral assumption, that the covariance term is zero. Thus we approximate the true demand equation with the aggregate equation (E1.5), while recognising that this involves an aggregation error $Cov_{Ti}[\eta_i, p_i]$.

Instead of assuming that the covariance term is zero, another way of obtaining equation (E1.5) is to make the more stringent assumption that the percentage change in price p_i is uniform across the group of commodities $i \in I$. In deriving the SALTER aggregation formulae below, we will often make analogous assumptions about the uniformity of variables within aggregation groups. The reader can verify that in each case, the same results could be obtained, at some cost in complexity, under the less stringent assumption that certain covariances between variables and parameters or coefficients from the database are zero. In each case, the procedure generates an aggregation error equal to the covariance.

Aggregation errors arising from restrictions on functional forms

As noted above, the SALTER theoretical structure imposes restrictions on functional forms for all demand systems. Typically, these restrictions involve the imposition of the CES functional form at various levels of the demand systems.

Unfortunately functional form is not generally preserved under aggregation. Consider for example the treatment of import sourcing in SALTER. For each commodity and use category, imports from different sources are combined by a CES sub-utility or sub-production function. When we aggregate the database, the corresponding functions in the aggregated model must also be CES. But this is liable to conflict with the objective of consistency in simulation results between the original and the aggregated model.

Suppose for example that we aggregate two commodities together. Suppose further that for some use category in some regions, imports are sourced initially as follows: commodity 1 is imported from regions 1 and 2, and commodity 2 from regions 3 and 4. Then to match the results generated by the original model, the aggregated model must allow imports from regions 1 and 2 to be substituted for each other, and likewise imports from regions 3 and 4; but must not allow for substitution between regions 1 and 3, 1 and 4, 2 and 3, or 2 and 4. Clearly this is impossible with a CES function.

This means that even when the zero covariance conditions are satisfied, the aggregated model cannot generally achieve consistency with the original model. Since consistency is unobtainable, our objective must therefore be to minimise the degree of inconsistency. To do this, we need some measure of inconsistency between the original and the aggregated model.

We can formulate the problem as follows. Suppose that for some demand system or subsystem, aggregation of the original model gives us a matrix of substitution elasticities $\mathbf{U} = [U_{i,k}]$, where $U_{i,k}$ denotes the elasticity of substitution between economic goods i and k. The SALTER theoretical structure constrains the matrix of substitution elasticities in the aggregated model to a one-parameter family $\mathbf{V}(\sigma)$, where σ denotes the elasticity of substitution in a CES function. If we can find some suitable measure $m(\mathbf{U}, \mathbf{V})$ of distance between matrices of substitution elasticities $\mathbf{U}$, $\mathbf{V}$, then we can choose the substitution elasticity σ so as to minimise the distance $m(\mathbf{U}, \mathbf{V}(\sigma))$ between the desired and the actual matrix.

The distance measure or *metric* should obviously vary directly with the difference $V_{i,k} - U_{i,k}$ between any pair of corresponding elements in the two matrices. Beyond this, the sensitivity of the metric to the various elements of the difference matrix should ideally reflect the importance given to results for the different goods in the demand system, and the importance of the various prices in the system as potential sources of changes in demands. A simple measure which reflects these considerations in a rough and ready way is the following:

$$m(\mathbf{V}, \mathbf{U}) = \sum_i \sum_k S_i S_k (V_{i,k} - U_{i,k})$$

where S_i denotes the share of good i in total expenditure allocated by this demand system. This measure appears suitable, because users are likely to place more importance on results for goods with high expenditure shares than on results for goods with low expenditure shares; and because changes in prices of goods with high expenditure shares will, other things equal, have more effect on demand for other goods than will changes in prices of goods with low expenditure shares.

Aggregation errors arising from cross-system restrictions

The SALTER theoretical structure imposes restrictions not only within demand systems but also between demand systems. In particular, it imposes uniform elasticities of substitution in intermediate usage across all industries in each region. Both the import-domestic substitution elasticity and the import-import substitution elasticity are uniform across industries.

Applying the procedure just described to intermediate usage by each industry, we would in general obtain a different preferred elasticity of substitution σ_J for each industry J in the aggregated model. We would then have to choose a common substitution elasticity σ as some compromise between the different σ_J.

We choose instead a somewhat different approach. We calculate from the original model the substitution elasticity matrix $\mathbf{U}_J$ for intermediate use by each industry J in the aggregated model; but then from the $\mathbf{U}_J$ and share coefficients we aggregate across all industries to calculate a matrix $\mathbf{U}$ of elasticities of substitution for total intermediate usage. Also from the share coefficients, we can determine the substitution elasticity matrix $\mathbf{V}(\sigma)$ in the aggregated model for total intermediate usage by all industries, as a function of the parameter σ. Then we choose that value of σ which minimises the discrepancy $m(\mathbf{U}, \mathbf{V}(\sigma))$, where as in the previous section m denotes a suitable metric.

In this procedure, the criterion for selecting the elasticity of substitution in the aggregated model is similarity in results for total intermediate usage by all industries, between the aggregated and original models.

3 Theoretical structure of the aggregation facility

This section outlines the techniques used to derive the parameter aggregation formulae. First, the aggregation of the input-output (*IO*) portion of the database is discussed. Second, the outcomes-oriented scheme is covered in some detail. Variations on the basic scheme are presented in Appendixes A to C. Finally, the process-oriented scheme is briefly discussed, and compared to the process-oriented scheme with an example.

A basic reference on aggregation for the ORANI model is Sutton (1981). We adapt and extend some of the techniques Sutton examined in that paper to the SALTER model.

Background on database aggregation

Notation is used to distinguish between aggregated and disaggregated quantities. Indices running over aggregated entities (ie. sources, industries) are denoted by capital letters, while indices on disaggregated entities are in lower case. For example, the notation $i \in I$ denotes a disaggregated commodity i that is a constituent of the aggregated commodity I. The formulae for most of the aggregated parameters are derived using database values and the disaggregated form of the equations. Levels values (ie. database values) are denoted by capital letters, while percent change variables are in lower case.

The IO portion of the database is aggregated using 'mapping matrices' created by the user in a FORTRAN program. These matrices list the correspondence between aggregated and disaggregated entities (see Section 4 for details). An aggregate value is formed by adding up the appropriate disaggregated values. For example, consider the value of consumption of domestically produced commodity *i* in region *z*, *DCON*(*i*,*z*). Given an aggregate commodity *I* and an aggregate region Z, the aggregate value of domestic consumption, $NDCON(I,Z)$, is obtained by summing over those commodities i that are constituents of the aggregate commodity I, and those regions z that are constituents of the aggregate region Z. The mathematical formula for this is :

$$NDCON(I,Z) = \sum_{i \in I} \sum_{z \in Z} DCON(i,z)$$

Outcomes-oriented aggregation

In outcomes-oriented aggregation, two preliminary assumptions are made in deriving the parameters for most of the aggregated equations. The first, which was mentioned in the previous section, is the preservation of all functional forms as they appear in the standard model. This assumption leads to difficulties in the case of the parameters appearing in CES functions. Additionally, it is assumed that the price and quantity variables in each of the disaggregated equations undergo the same percentage changes as the corresponding aggregated variables. This assumption is expressed more precisely in equation (A2) below. Given a particular model application, users should remember these assumptions in selecting an aggregation of industries/commodities and destinations/sources.

In what follows, three examples from the consumer behaviour section of the model illustrate the derivations of aggregation formulae. The CES example illustrates why consistency in aggregation cannot normally be obtained, and how the cross-system restrictions built into the model structure are handled.

The CES aggregation example chosen is the equation defining consumer demand for imported commodities by source, equation (S16) in Jomini *et al.* (1991). The notation used here follows the notation from that document, with several exceptions. The superscript I appearing on all coefficients and variables and the subscript c on prices have been dropped for simplicity. The elasticity of substitution between imports from different sources is denoted by β_i^{Mz}, the M indicating we are considering an import substitution parameter. In the context of consumer equations, all prices that appear should be understood to be consumer prices.

The disaggregated equation (S16) is

$$c_{i,s}^{z} = c_{i}^{z} - \beta_{i}^{Mz}(p_{i,s}^{z} - p_{i}^{z}) \qquad \text{for all } i, s, z \qquad (1)$$

where

$$p_{i}^{z} = \sum_{m} S_{i,m}^{z} \cdot p_{i,m}^{z} \qquad (2)$$

The sum in (2) is over all sources m, and $S_{i,m}^{z}$ is the share of consumption in region z of the imported variety of commodity i from source m in total consumption of commodity i in region z. The values relevant to the consumption of imports are $ICONS(i,z,s)$ and $TCRIS(i,z,s)$, the level of consumption and corresponding taxes (respectively) in region z on imports of commodity i from source s. So,

$$S_{i,m}^{z} = \frac{ICONS(i,z,m) + TCRIS(i,z,m)}{\sum_{s}(ICONS(i,z,s) + TCRIS(i,z,s))}$$

Let $ICS(i,z,s)$ denote the value at purchasers' prices of the consumption in region z of imported commodity i from source s, so :

$$ICS(i,z,s) = ICONS(i,z,s) + TCRIS(i,z,s)$$

The corresponding aggregated value is :

$$NICS(I,Z,S) = \sum_{i \in I} \sum_{z \in Z} \sum_{s \in S}(ICONS(i,z,s) + TCRIS(i,z,s))$$

The assumption that we preserve the CES format of the aggregated equation gives

$$c_{I,S}^{Z} = c_{I}^{Z} - \beta_{I}^{MZ}(p_{I,S}^{Z} - p_{I}^{Z}) \qquad \text{for all } I, S, Z \qquad (A1)$$

where

$$p_{I}^{Z} = \sum_{N} S_{I,N}^{Z} \cdot p_{I,N}^{Z}$$

and the sum is taken over aggregated sources N.

The assumption that disaggregated prices and quantities experience the same percent changes as the corresponding aggregated prices and quantities implies

$$p_{i,s}^{z} = p_{I,S}^{Z} \text{ for all } i \in I, z \in Z, s \in S \qquad (A2)$$

and

$$c_i^z = c_I^Z \text{ for all } i \in I, z \in Z$$

Clearly, the simplifying assumptions in (A2) involve a loss of information.

Our aim now is to find a value for the aggregate substitution elasticity β_I^{MZ} in terms of database IO values and disaggregated parameters.

The first step is to derive the relationship between the aggregate equation for import consumption demand and the constituent disaggregated equations. We know we can express the value of consumption as the product of a price and a quantity:

$$I_{i,s}^z = P_{i,s}^z \cdot C_{i,s}^z = ICS(i,z,s)$$

where $I_{i,s}^z$ denotes consumption expenditure on imported commodity i from source s in region z, and $P_{i,s}^z$ and $C_{i,s}^z$ are the corresponding price and quantity. Fixing some I, Z and S, the corresponding aggregate is a simple sum :

$$I_{I,S}^Z = \sum_{i \in I} \sum_{s \in S} \sum_{z \in Z} P_{i,s}^z \cdot C_{i,s}^z = NICS(i,z,s) \qquad \text{(L1)}$$

Expressing the preceding relationship in differential (percent change) form, we find

$$i_{I,S}^Z = \sum_{i \in I} \sum_{s \in S} \sum_{z \in Z} W_{i,s}^z (p_{i,s}^z + c_{i,s}^z) \text{ for all } i \in I, s \in S, z \in Z \qquad (3)$$

where

$$W_{i,s}^z = \frac{ICS(i,s,z)}{NICS(I,Z,S)} \text{ for all } i \in I, z \in Z, s \in S$$

Pearson and Codsi (1991) illustrate the details of expressing a levels equation in percent change form.

The result of the previous derivation is that the aggregated equation can be expressed as a weighted average of its (disaggregated) constituent equations. Most parameters in the aggregated system are derived in this manner, a notable exception being the parameters in the uppermost level of the consumer demand system.

The natural definition of $c_{I,S}^Z$ is:

$$c_{I,S}^{Z} = \sum_{i\in I}\sum_{s\in S}\sum_{z\in Z} W_{i,s}^{z} \cdot c_{i,s}^{z} \quad (3a)$$

We now rewrite equation (3a) by substituting the right hand side of equation (1) into the right hand side of (3a). Also, we replace the expression for p_i^z by its definition in terms of a share-weighted sum:

$$c_{I,S}^{Z} = \sum_{i\in I}\sum_{s\in S}\sum_{z\in Z} W_{i,s}^{z}[c_i^z - \beta_i^{Mz}(p_{i,s}^{z} - \sum_{m} S_{i,m}^{z} \cdot p_{i,m}^{z})] \quad (4)$$

Finally, we substitute the price and quantity assumptions from (A2) into the right hand side of (4) :

$$c_{I,S}^{Z} = \sum_{i\in I}\sum_{s\in S}\sum_{z\in Z} W_{i,s}^{z}[c_I^Z - \beta_i^{Mz}(p_{I,S}^{Z} - \sum_{m} S_{i,m}^{z} \cdot p_{I,N}^{Z})] \quad (5)$$

Note that

$$\sum_{i\in I}\sum_{s\in S}\sum_{z\in Z} W_{i,s}^{z} = 1$$

Since the denominators of the share terms $S_{i,m}^{z}$ are independent of source, for each aggregate source N we can group the constituent $m \in N$ as

$$S_{i,N}^{z} = \sum_{m\in N} S_{i,m}^{z}$$

Implementing this substitution, (5) becomes

$$c_{I,S}^{Z} = c_I^Z + \sum_{i\in I}\sum_{s\in S}\sum_{z\in Z} W_{i,s}^{z} \cdot \beta_i^{Mz}(p_{I,S}^{Z} - \sum_{N} S_{i,N}^{z} \cdot p_{I,N}^{Z}) \quad (5a)$$

If we implement the assumption of maintenance of CES format (A1) in the left hand side of (5a), we see:

$$c_I^Z - \beta_I^{MZ}(p_{I,S}^{Z} - \sum_{N} S_{I,N}^{Z} \cdot p_{I,N}^{Z}) = c_I^Z + \sum_{i\in I}\sum_{s\in S}\sum_{z\in Z} W_{i,s}^{z} \cdot \beta_i^{Mz}(p_{I,S}^{Z} - \sum_{N} S_{i,N}^{z} \cdot p_{I,N}^{Z}) \quad (6)$$

Equating coefficients on each of the price terms, we see

$$\beta_I^{MZ} = \sum_{i \in I} \sum_{s \in S} \sum_{z \in Z} \frac{(W_{i,s}^z \cdot S_{i,N}^z \cdot \beta_i^{Mz})}{S_{I,N}^Z} \text{ for } N \neq S \tag{C1}$$

and

$$\beta_I^{MZ} = \sum_{i \in I} \sum_{s \in S} \sum_{z \in Z} \frac{W_{i,s}^z \cdot (1 - S_{i,S}^z)}{(1 - S_{I,S}^Z)} . \beta_i^{Mz} \tag{C2}$$

Denote the right hand sides of (C1) and (C2) as $\beta_{I,(N,S)}^{MZ}$, where N and S run over all aggregated sources. If there are K aggregated sources, then (C1) and (C2) describe K^2 aggregated parameters β_I^{MZ}, $K(K-1)/2$ of which are distinct.

Consider the N by S matrix of these aggregated 'elasticities',

$$\left[\beta_{I,(N,S)}^{MZ}\right]_{(N,S)} = \mathbf{D}$$

Each column of **D** corresponds to an aggregated consumer demand equation specified for source S by (C1) and (C2). Define a matrix **E**,

$$\mathbf{E} = \left[\beta_I^{MZ}\right]_{(N,S)}$$

each entry of which is the aggregate elasticity β_I^{MZ}.

Our aim of preserving functional form means that we need to find a parameter β_I^{MZ} such that the matrix **E** is in some sense as close as possible to **D**.

To obtain a value for β_I^{MZ} that satisfies this aim, we chose to minimise a loss function of the form

$$f(\beta_I^{MZ}) = \sum_S \sum_N AW_I^Z(N,S)(\beta_I^{MZ} - \beta_{I,(N,S)}^{MZ})^2 \tag{7}$$

The weighting coefficients $AW_I^Z(N,S)$ are defined by the formula

$$AW_I^Z(N,S) = S_{I,N}^Z \cdot S_{I,S}^Z$$

Since f is quadratic, we are assured a unique minimum value, found by solving

$$\frac{df}{d\beta_I^{MZ}} = 0 \tag{8}$$

for β_I^{MZ}. Explicitly, equation (8) is

$$0 = 2 \cdot \sum_S \sum_N AW_I^Z(N,S)(\beta_I^{MZ} - \beta_{I,(N,S)}^{MZ}) \tag{9}$$

Solving (9) for β_I^{MZ} and noting that

$$\sum_S \sum_N AW_I^Z(N,S) = 1$$

we find that

$$\beta_I^{MZ} = \sum_S \sum_N AW_I^Z(N,S)\beta_{I,(N,S)}^{MZ} \tag{10}$$

In the coefficients $AW_I^Z(N,S)$, each index S corresponds to a different aggregate equation (see equation (A1)), and the indices N correspond to the values defined by (C1) and (C2) for this S. Thus, the larger the share a source S has in the consumption of (imported) commodity I in region Z, the closer the final value of β_I^{MZ} will be to the constraints defined by S.

The correspondence between the formulation of β_I^{MZ} in (10) and the expression in the TABLO code is given in Appendix A.

Equations (S25) and (S41) in Jomini *et al.* (1991), investment demand and government demand for imported commodities by source, respectively, are aggregated in an identical manner to (S16). Of course, there are obvious modifications in notation and terminology (ie. substitute 'investment' or 'government' for 'consumption'). Appendix B gives the derivation of the CES parameter in equation (S18), industry demands for primary factors.

Although we have chosen a deterministic approach to estimating values for this class of CES parameters, a stochastic method suggested by Kevin Hanslow is worth noting here. In his approach, the disaggregated prices and quantities are assumed to have a normal distribution, whose variance/covariance matrix is calculated by applying appropriate random shocks to the model. The aggregated CES parameter is found by maximising a likelihood function. The value for the parameter will then depend on the shocks chosen to calculate the price and quantity distributions.

Our second derivation is for the elasticity of substitution between domestic and imported commodities, β_i^z. The relevant equations in Jomini *et al.* (1991) are

$$c_i^{Dz} = c_i^z - \beta_i^z (p_i^{Dz} - p_i^z) \qquad \text{(S14)}$$

$$c_i^{Mz} = c_i^z - \beta_i^z (p_i^{Mz} - p_i^z) \qquad \text{(S15)}$$

where

$$p_i^z = S_i^{Dz} p_i^{Dz} + S_i^{Mz} p_i^{Mz}$$

If we let

$$D_i^z = DCONS(i,z) + TCRD(i,z)$$

and

$$M_i^z = \sum_s ICONS(i,z,s) + \sum_s TCRIS(i,z,s)$$

then

$$S_i^{Dz} = \frac{D_i^z}{D_i^z + M_i^z} \text{ and } S_i^{Mz} = \frac{M_i^z}{D_i^z + M_i^z}$$

We can express (S14) in a slightly modified form that is better suited to our purposes:

$$c_i^{Dz} = c_i^z - \beta_i^z \cdot S_i^{Mz} (p_i^{Dz} - p_i^{Mz}) \qquad \text{(S14')}$$

Equation (S15) can be similarly modified. Although the derivation proceeds using (S 14 '), identical results are obtained by starting with equation (S14).

Our assumptions on maintaining functional form and the movement of disaggregated prices and quantities are

$$c_I^{DZ} = c_I^Z - \beta_I^Z \cdot S_I^{MZ} (p_I^{DZ} - p_I^{MZ}) \qquad \text{(A3)}$$

and

$$p_i^{Dz} = p_I^{DZ}$$
$$p_i^{Mz} = p_I^{MZ}$$

and

$$c_i^z = c_I^Z$$

for all $i \in I$, $z \in Z$. We can express the aggregated equation as a weighted average of the disaggregated constituent equations. The weights D_i^z / D_I^Z arise, as in equation (3), by differentiating an identity in levels values. Implementing our assumptions (A3) and (A4), we find

$$\begin{aligned} & p_I^{DZ} + c_I^Z - \beta_I^Z \cdot S_I^{MZ} (p_I^{DZ} - p_I^{MZ}) \\ & = \sum_{i \in Z} \sum_{z \in Z} \frac{D_i^z}{D_I^Z} \left[p_I^{DZ} + c_I^Z - \beta_i^z \cdot S_i^{Mz} (p_I^{DZ} - p_I^{MZ}) \right] \end{aligned} \qquad (11)$$

Cancelling like terms and solving for β_I^Z, the formula for the aggregated parameter is:

$$\beta_I^Z = \frac{\sum_{i \in I} \sum_{z \in Z} \frac{D_i^z \cdot M_i^z}{(D_i^z + M_i^z)} \cdot \beta_i^z}{\frac{D_I^Z \cdot M_I^Z}{(D_I^Z + M_I^Z)}}$$

Notice that this expression is symmetric with respect to imported and domestic values. The TABLO formula notation for β_I^Z is nearly identical to the notation used in this text. Appendix D contains the explicit correspondence in notation.

Aggregate versions of equation sets (S39)-(S40) and (S23)-(S24), respectively government and investment demands, follow the previous derivation, with the obvious changes of notation and wording.

The equations describing intermediate demands for inputs, (S20) - (S22) in Jomini *et al.* (1991), have the added complication of an industry dimension. The derivation of the associated parameters, however, is a straightforward variation on the techniques illustrated in the first two examples. Further details are provided in Appendix C.

The next derivation is for equation (S13) which describes the consumer demand for commodity aggregates via the linear expenditure system (LES):

$$c_i^z = \sum_{h=1}^{I} \lambda_{i,h}^z \cdot p_{Ch}^z + \mu_i^z \cdot c_T^z \qquad (S13)$$

Our only assumption here is that the aggregated equation has the same format as the disaggregated.

The parameters appearing in this equation are μ_i^z, the elasticity of demand for commodity i in region z with respect to aggregate consumption expenditure, and $\lambda_{i,j}^z$, the price elasticity of demand for i with respect to the price of commodity j in region z. Two related parameters are *FRISCH*(z), the Frisch parameter in region z, and *MBSHARE*(i,z), the marginal budget share of commodity i in region z. The relationships between these four parameters are

$$\mu_i^z = \frac{MBSHARE\ (i,z)}{BSHARE\ (i,z)}$$

and

$$\lambda_{i,j}^z = -\mu_i^z \cdot BSHARE\ (j,z) \cdot \left(1 + \frac{\mu_j^z}{FRISCH\,(z)}\right) + \delta_{i,j} \cdot \frac{\mu_i^z}{FRISCH\,(z)}$$

where *BSHARE*(i,z) is the share of consumption expenditure in region z on commodity i (from all sources) in total expenditure in z, and $\delta_{i,j}$ is the Kronecker delta. *BSHARE*(i,z) is derived from database values.

Examining these relationships, it is clear that if we can define the aggregated parameters *MBSHARE*(I,Z) and *FRISCH*(Z), then we can deduce values for the aggregated parameters μ_I^Z and $\lambda_{I,J}^Z$.

As a beginning, we need to define some new terms. Expenditure on consumption can be expressed as the sum of expenditure on necessities and 'supernumerary expenditure'. The 'supernumerary ratio' is defined as the quotient of supernumerary expenditure over consumption expenditure. In the context of the SALTER model, these quantities are all region-specific. The Frisch parameter in the LES is identified as the negative reciprocal of the supernumerary ratio (see Powell 1974, pp. 36-41). If we let *SN*(z) denote the supernumerary ratio in region z and CTT(z) denote consumption expenditure in region z, then

$$SN\,(z) = \frac{CTT\,(z)}{-FRISCH\,(z)}$$

and

$$SN(Z) = \sum_{z \in Z} SN(z)$$

In aggregate form, the Frisch parameter becomes:

$$FRISCH(Z) = \frac{\sum_{z \in Z} CTT(z)}{\left(-\sum_{z \in Z} SN(z) \right)} = \frac{CTT(Z)}{SN(Z)}$$

The marginal budget shares also have a simple economic interpretation in the LES (see Goldberger 1987, p.46). *MBSHARE*(i,z) is the proportion of the supernumerary expenditure spent on good i in region z. So

$$\sum_{i} MBSHARE\ (i, z) = 1$$

Define a new levels quantity,

$$SNI\ (i, z) = MBSHARE\ (i, z) \cdot SN(z)$$

ie. *SNI*(i,z) is the share of supernumerary expenditure on good i in region z. In aggregated form,

$$SNI\ (I, Z) = MBSHARE\ (I, Z) \cdot SN(Z)$$

But, we can also express the share of supernumerary expenditure in Z on aggregated commodity I as a simple sum:

$$SNI\ (I, Z) = \sum_{i \in I} \sum_{z \in Z} MBSHARE\ (i, z) \cdot SN(z)$$

Equating these two expressions for *SNI*(I,Z) and solving for the aggregate marginal budget share, we find

$$MBSHARE\ (I, Z) = \frac{\sum_{z \in Z} SN(z) \cdot \left(\sum_{i \in I} MBSHARE\ (i, z) \right)}{\sum_{z \in Z} SN(z)}$$

So, in the case of the LES, we departed from the standard approach of expressing the aggregate equation as a weighted sum of the disaggregated equations.

The aggregation formulae for the remaining two parameters appearing in the model turn out to have the same form assumed in the 'process-oriented' aggregation scheme.

One of these, H3(*Z*), the indexing of wages to the CPI in region *Z*, only appears in an equation in the TABLO implementation of the model. The aggregation formula for the parameter is derived, as in equation (3), by differentiating an identity in level values.

The final parameter appearing in the model is the elasticity of labour supply in region *z*, χ_1^Z. It appears in equation (S17) in Jomini *et al.* (1991):

$$f_{S1}^z = \chi_1^z (w_1^z - cpi^z - R_{TY}^z t_Y^z) \qquad \text{(S17)}$$

where R_{TY}^z is the ratio of the database values *TAXLY*(*z*) over *HHDISPY*(*z*), or tax on labour over disposable income. Define *LTT*(*z*) to be the (levels) usage of labour in region *z*. Implementing our standard assumptions, we can express our aggregation relationship as

$$\chi_1^Z (w_1^Z - cpi^Z - R_{TY}^Z \cdot t_Y^Z) = \sum_{z \in Z} \frac{LTT(z)}{LTT(Z)} \left[\chi_1^z (w_1^Z - cpi^Z - R_{TY}^z t_Y^Z) \right] \qquad (12)$$

Equating coefficients on w_1^z and cpi^Z, we obtain

$$\chi_1^Z = \sum_{z \in Z} \frac{LTT(z)}{LTT(Z)} \cdot \chi_1^z \qquad (13)$$

while on t_y^Z, we obtain

$$\chi_1^Z = \sum_{z \in Z} \frac{LTT(z) \cdot R_{TY}^z}{LTT(Z) \cdot R_{TY}^Z} \cdot \chi_1^z \qquad (14)$$

To reconcile the discrepancy between values indicated in (13) and (14), we will assume that:

$$\frac{R_{Ty}^{z}}{R_{Ty}^{Z}} = 1 \text{ for each } z \in Z$$

Thus we accept the value of the aggregated parameter χ_I^Z given in equation (13).

Process oriented aggregation

In the process-oriented aggregation scheme, the derivations of the parameter formulae are straightforward. The derivations all proceed in the same manner as that of equation (3). In short, an identity in levels values (see (L1)) is differentiated and placed in percent change form. The weighting factors on the disaggregated equations are then used as weights on the corresponding diasaggregated parameters. An example of a formula of this type is equation (13) above. The discussion in Section 2 presents a more detailed exposition on process-oriented aggregation.

An example

In this final subsection, we compare results obtained in each of the two aggregation schemes for a particular example. The sample aggregation took a 37 commodity, 9 region version of the database and produced a final seven commodity, three region database.

Tables 1 and 2 list the results for the parameter β_I^{MZ} occurring in equation (1) above. The results show that the values obtained in the outcomes-oriented scheme are somewhat lower than in the process-oriented scheme. The differences observed between the two schemes may be lessened if, for example, commodities are highly aggregated, or aggregated with no regard to the parameter settings.

Table 1: **Process-oriented aggregation**

Industries							
Regions	1	2	3	4	5	6	7
1	4.40	5.60	6.20	8.80	3.60	3.80	10.40
2	4.40	5.60	6.20	8.80	3.60	3.80	10.40
3	4.40	5.60	6.20	8.80	3.60	3.80	10.40

Table 2: **Outcomes-oriented aggregation**

Industries							
Regions	1	2	3	4	5	6	7
1	3.81	4.92	5.83	8.52	2.84	3.19	10.18
2	3.46	5.24	4.84	8.37	3.38	3.55	10.35
3	3.72	5.13	5.99	8.73	3.26	2.98	9.25

4 Implementation and application of the aggregation facility

The group of facilities covered in this section implement the outcomes-oriented and process-oriented aggregation schemes. The facilities allow the user to choose an industry only, region only, or industry plus regional aggregation. You can also use the programs merely to change the ordering of the database.

In the outcomes-oriented aggregation scheme, parameters are aggregated so as to preserve the theoretical structure of the disaggregated model as closely as possible. In the process-oriented aggregation scheme, the aggregated parameters are expressed as simple share-weighted sums of the appropriate disaggregated parameters. The latter is the form that should be used when the database is merely being reordered.

A brief description of the use of each of the facilities follows. The GEMPACK documentation should be consulted for detailed instructions on the use of TABLO programs.

AGGMAP.FOR is a FORTRAN program that produces mapping matrices. The matrices list the correspondence between the aggregated, or rearranged, industries and/or regions and the original disaggregated ones. The program can be run either interactively or in batch mode. When using AGGMAP.FOR interactively, the user should have available a list of the current industry and regional arrangements along with the specification of an aggregation. A stored input (*stinp*) file is required for use in batch mode.

The output of the program is contained in a header array file specified by the user. It is helpful to give the file a descriptive name such as, for example, 'agg75.map' for a final seven commodity, five region aggregation. The file contains two matrices, giving the regional (AGGR) and industry (AGG) aggregation mappings, plus headers containing the number of old and new industries and regions.

Two steps are needed to create a useable program out of the FORTRAN code. First, the code is compiled by typing 'COMPTB AGGMAP'. Then, the program is linked to the 'libraries' by typing ' AGGMAP'.

At this point an executable program exists. It is run interactively by typing 'UP AGGMAP'. The user then creates the mapping matrices by responding to self-explanatory prompts. To run AGGMAP in batch mode, type in:

'UP AGGMAP < ??.STI > OUT'

OUT is a typical user specified name for the file containing the screen output of the program, while ??.STI should be replaced with the name of a stinp deck. The stinp deck **'AGG.STI'** gives a sample aggregation of a 37 commodity, 9 region version of the SALTER database (see Appendix E). It also contains explanatory comments indicating how it can be altered for individual use.

AGGPO.TAB is a TABLO program that produces an aggregated database using the process-oriented parameter aggregation scheme. There are two inputs into this program. The name of the original database is entered in response to the prompt for the file corresponding to 'DATIO'. The name of the mapping file produced by the program AGGMAP.FOR is entered in response to the prompt for 'AGGREG'. The name chosen for the output file is given in response to the prompt for the file 'AGGDAT'.

To run this program, type in 'UP AGGPO'.

AGGIO.TAB and **AGGCES.TAB** are TABLO programs that create an aggregated database using the outcomes-oriented scheme for parameter aggregation. As the aggregation formulae for four of the CES parameters are complicated and require many coefficients, their computation is done in the program AGGCES.TAB. The rest of the parameters, along with the IO portion of the database, are aggregated in AGGIO.TAB.

In both of these TABLO programs, the user enters the name of the original database in response to the file corresponding to 'DATIO', and the name of the file produced by AGGMAP.FOR in response to the prompt for 'AGGREG'. The user enters a (different) name for the output file of each of the programs in response to the prompt for the file 'NDAT'.

ADDCES.STI is a stinp deck used to combine the output files produced by AGGIO.TAB and AGGCES.TAB into a single header array file. The stinp deck is fed into the TABLO program MODHAR. It should be examined and altered to give the correct input and output file names. Descriptive comments appear to the right of the instructions that are read in by MODHAR at each step. Appendix E contains a copy of this file.

Appendix A: Tablo correspondence for the import consumption parameter

The parameter β_J^{MZ} is the elasticity of substitution between sources of imported commodity J used for consumption in region Z. It appears in equation (S16) in Jomini *et al.* (1991).

In this appendix, we explicitly define the correspondence between the aggregation formula of β_J^{MZ} given in this text and in the TABLO implementation. The formula appearing in the text is:

$$\beta_J^{MZ} = \sum_S \sum_N AW_J^Z(N,S) \cdot \beta_{J,(N,S)}^{MZ} \tag{15}$$

where

$$AW_J^Z(N,S) = S_{J,N}^Z \cdot S_{J,S}^Z$$

In the definitions of $\beta_{J,(N,S)}^{MZ}$ given in equations (C1) and (C2) of Section 3, there are different forms for the cases $N = S$ and $N \neq S$. We can split the sum in (15) along these lines:

$$\beta_J^{MZ} = \sum_S \sum_{N,N\neq S} AW_J^Z(N,S) \cdot \beta_{J,(N,S)}^{Mz} + \sum_S AW_J^Z(S,S) \cdot \beta_{J,(S,S)}^{MZ} \tag{16}$$

Call the left-hand term the 'cross sum' and the right-hand term the 'own sum'. Now, we indicate the correspondence with the TABLO implementation of the cross sum. The parameter, β_J^{MZ}, is known as *NBETAI(J,Z)*. The summation terms in (16) appear as:

NBETAI(J,Z) = SUM(*S*, *NEWREG*, SUM(*N*, *NEWREG*: *OD(S,N)* = 1,...)

+ SUM(*S*, *NEWREG*,...).

OD(S,N) is a matrix with *OD(S,N)*=1 when $S \neq N$ and *OD(S,S)*=0 for each *S*. In the cross sum, the conditional ': *OD(S,N)* = 1' indicates we only sum over those terms where $S \neq N$.

Fix some pair *(N,S)* such that $N \neq S$. In the notation of Section 3 of this text:

$$AW_J^Z \cdot \beta_{J,(N,S)}^{MZ} = S_{J,N}^Z \cdot S_{J,S}^Z \left[\sum_{i \in J} \sum_{s \in S} \sum_{z \in Z} \left(\frac{W_{i,s}^z \cdot S_{i,N}^z \cdot \beta_i^{Mz}}{S_{J,N}^Z} \right) \right] \tag{17}$$

We can express everything inside the square brackets as

$$\sum_{i \in J} \sum_{z \in Z} \frac{S_{i,N}^Z \cdot \left(\sum_{s \in S} W_{i,s}^z \right) \cdot \beta_i^{Mz}}{S_{J,N}^Z} \tag{18}$$

since all terms other than $W_{i,s}^z$ are independent of source S.

Now, we give the correspondence between the coefficients listed in (18) with the TABLO code:

$S_{J,N}^Z = ACS(J,N,Z)$ (ACS = Aggregate Consumption Share)

$1/S_{J,N}^Z = IACS(J,N,Z)$ ($IACS$ = Inverse Aggregate Consumption Share)

$S_{i,N}^z = CSAS(i,N,z)$ ($CSAS$ = Consumption Share Aggregated Source)

and

$$\sum_{s \in S} W_{i,s}^z = MCONAS(i,z,s) \cdot IAC(J,Z,S)$$

($MCONAS$ = Import Consumption, Aggregated Source)
(IAC = Inverse Aggregate Consumption)

Now, we can fully express the TABLO code of the cross sum:

NBETAI(J,Z) = SUM(*S*, *NEWREG*, SUM(*N*, *NEWREG*: *OD*(*S*,*N*)=1,
ACS(*J*,*Z*,*S*)**ACS*(*J*,*Z*,*N*)*SUM(*i*, *IND*: *AGG*(*i*,*J*)=1,
SUM(*y*, *REG*: *AGGR*(*y*,*Z*)=1, *MCONAS*(*i*,*y*,*S*)**IAC*(*J*,*Z*,*S*)*
CSAS(*i*,*y*,*N*)**IACS*(*J*,*Z*,*N*)**BETAI*(*i*,*y*)))))+..... (19)

Given a particular pair (*J*,*Z*), the two conditional sums in (19),

SUM(*i*, *IND*: *AGG*(*i*,*J*)=1, SUM(*y*, *REG*: *AGGR*(*y*,*Z*)=1,...)

only sum over those commodities i that are constituents of the aggregate commodity J, and those constituent regions y of the aggregate region Z.

For an aggregate source S in the 'own' sum, we have

$$AW_J^Z \cdot \beta_{J,(S,S)}^{MZ} = S_{J,S}^Z \cdot S_{J,S}^Z \left[\sum_{i \in J} \sum_{s \in S} \sum_{z \in Z} \left(\frac{W_{i,s}^z \cdot (S_{i,S}^z - 1) \cdot \beta_i^{Mz}}{(S_{J,S}^Z - 1)} \right) \right] \tag{20}$$

Two coefficients need to be defined in the TABLO code for the 'own' sum:

$$\left(S_{i,S}^y - 1\right) = (CSAS\,(i,y,S) - 1)$$

and

$$1/\left(S_{J,S}^Z - 1\right) = IACDS(J,Z,S)$$

($IACDS$ = Inverse Aggregate Consumption Difference Share)

The sign changes, in comparison with the formulae that appear in this text, are irrelevant since these two coefficients are multiplied.

Appendix B: Primary factor demand

Equation (S18) in Jomini *et al.* (1991) describes industry demands for primary factors. The parameter aggregation procedure essentially follows the derivation in Section 3 for CES parameters. In particular, we assume the maintenance of functional form and that disaggregated prices and quantities move in line with their corresponding aggregates. In this appendix, we additionally assume that the disaggregated shift terms, the 'a' terms, experience the same percent change as the aggregated ones (see assumption A5, below). The relevant equations and assumptions are

$$f_{k,j}^{z} = q_{j}^{z} - \sigma_{j}^{z}\left[w_{k,j}^{z} - \sum_{m=1}^{k} S_{m,j}^{z} w_{m,j}^{z} \right] + a_{j}^{z} + a_{k,j}^{z} - \sigma_{j}^{z}\left[a_{k,j}^{z} - \sum_{m=1}^{k} S_{m,j}^{z} a_{m,j}^{z} \right] \quad \text{(S18)}$$

and

$$q_{j}^{z} = q_{J}^{Z},\ a_{j}^{z} = a_{J}^{Z},\ \text{for all } j \in J, z \in Z \quad \text{and} \quad \text{(A5)}$$

$$w_{k,j}^{z} = w_{k,J}^{Z},\ q_{k,j}^{z} = q_{k,J}^{Z},\ a_{k,j}^{z} = a_{k,J}^{Z},\ \text{for all } j \in J, z \in Z$$

As usual, we assume the aggregate version of (S18) maintains the same functional form. Note that primary factors are never aggregated, and we use the letters *k* and *m* for primary factor indices.

Following the steps given in the consumption example, we again find that the aggregate parameter σ_{J}^{Z} is subject to conditions. Fixing some *k*, *J* and *Z*, the conditions are

$$\sigma_{J,(k,m)}^{Z} = \sum_{j \in J} \sum_{z \in Z} \frac{FW_{j,z}^{k} \cdot S_{m,j}^{z}}{S_{m,J}^{Z}} \cdot \sigma_{j}^{z} \text{ for } m \neq k \quad \text{(C3)}$$

and

$$\sigma_{J,(k,k)}^{Z} = \sum_{j \in J} \sum_{z \in Z} \frac{FW_{j,z}^{k}(S_{k,j}^{z} - 1)}{(S_{k,J}^{Z} - 1)} \cdot \sigma_{j}^{z} \quad \text{(C4)}$$

Note that only double sums are necessary in (C3) and (C4), compared with triple sums in (C1) and (C2), since factors are not aggregated. $S^{Z}_{k,j}$ is the share of use of primary factor *k* in industry *j* in region *z*, in total primary factor usage (in *j*, in *z*) and $FW^{k}_{j,z}$ is the ratio of usage of factor *k*, in *j*, in *z* over the corresponding aggregate.

Here the conditions are determined by the primary factors. The value of the aggregate parameter is found by minimising a loss function in σ^{Z}_{J}, as in equations (8) – (10) in Section 3. In the present case, each sum is taken over the three primary factors, and the weights in the loss function, $AFW^{Z}_{J}(m,k)$, are the products of aggregate primary factor shares, $S^{Z}_{k,J}$ and $S^{Z}_{m,J}$. The formula for σ^{Z}_{J} is then found to be

$$\sigma^{Z}_{J} = \sum_{m}\sum_{k} AFW^{Z}_{J}(m,k) \cdot \sigma^{Z}_{J,(m,k)}$$

where the parameters $\sigma^{Z}_{J,(m,k)}$ are given by the constraints defined in (C3) and (C4).

Appendices A and D contain details on the TABLO code correspondence with the formula for σ^{Z}_{J}.

Appendix C: Intermediate use

This section contains brief derivations of the parameter aggregation schemes for equations specifying intermediate demand for imported and domestic commodities. The techniques are similar to those developed in Section 3, but with the added complication of an industry dimension.

First, we consider the demand for imported intermediates by source:

$$x_{i,s,j}^{Mz} = x_{i,j}^{Mz} - \eta_i^{Mz}\left(p_{i,s,j}^{Mz} - p_{i,j}^{Mz}\right) \qquad \text{(S22)}$$

Fix some aggregate *I*, *J*, *S* and *Z*. In levels values, we can express usage as a price times a quantity:

$$(IINTS(I,J,Z,S) + TRIS(I,J,Z,S))$$

$$= P_{I,S,J}^{MZ} . X_{I,S,J}^{MZ}$$

$$= \sum_{i \in I} \sum_{j \in J} \sum_{z \in Z} \sum_{s \in S} P_{i,s,j}^{Mz} . X_{i,s,j}^{Mz} \qquad (21)$$

Differentiate (21) and express in per cent change form:

$$p_{i,s,j}^{Mz} + x_{i,s,j}^{Mz} = \sum_{i \in I} \sum_{j \in J} \sum_{z \in Z} \sum_{s \in S} W_{i,s,j}^{Mz}\left(p_{i,s,j}^{Mz} + x_{i,s,j}^{Mz}\right) \qquad (22)$$

Each weight, $W_{i,s,j}^{Mz}$, is the ratio of disaggregated intermediate usage over the corresponding aggregate. The sum of the weights is one. If we implement in (22) the assumptions of maintaining CES format in the aggregate, and the following price and quantity assumptions:

$$p_{i,s,j}^{Mz} = p_{I,S,J}^{MZ} \text{ for all } i \in I,\ j \in J,\ z \in Z,\ s \in S$$

and

$$x_{i,j}^{Mz} = x_{I,J}^{MZ} \text{ for all } i \in I,\ j \in J,\ z \in Z$$

we find:

$$p_{I,S,J}^{MZ} + x_{I,J}^{MZ} - \eta_I^{MZ}\,(p_{I,S,J}^{MZ} - p_{I,J}^{MZ})$$

$$= \sum_{i \in I} \sum_{j \in J} \sum_{z \in Z} \sum_{s \in S} W_{i,s,j}^{Mz} \left[p_{I,S,J}^{MZ} + x_{I,J}^{MZ} - \eta_i^{MZ} \left(p_{I,S,J}^{Mz} - p_{i,j}^{Mz} \right) \right] \tag{23}$$

Recall that $p_{I,J}^{MZ}$ and $p_{i,j}^{Mz}$ are share-weighted sums of the appropriate source specific prices.

Equating coefficients on the price terms and implementing suggestive notation on the aggregate parameters we find:

$$\eta_{I,J,(S,S)}^{MZ} = \sum_{i \in I} \sum_{j \in J} \sum_{z \in Z} \sum_{s \in S} W_{i,s,j}^{Mz} \left[\frac{\left(1 - S_{i,S,j}^{Mz}\right) \cdot \eta_i^{Mz}}{\left(1 - S_{I,S,J}^{MZ}\right)} \right]$$

$$\eta_{I,J,(S,N)}^{MZ} = \sum_{i \in I} \sum_{j \in J} \sum_{z \in Z} \sum_{s \in S} \frac{W_{i,s,j}^{Mz} \cdot S_{i,N,j}^{Mz} \cdot \eta_i^{Mz}}{S_{I,N,J}^{MZ}} \quad \text{for } S \neq N$$

As in Section 3, we find the value for our aggregate parameter by minimising a loss function:

$$f\left(\eta_I^{MZ}\right) = \sum_J \sum_S \sum_N AW_I^Z(J,S,N) \left(\eta_I^{MZ} - \eta_{I,J,(S,N)}^{MZ} \right)^2$$

The aggregate parameter is found to be:

$$\eta_I^{MZ} = \sum_J \sum_S \sum_N AW_I^Z(J,S,N) \cdot \eta_{I,J,(S,N)}^{MZ}$$

The weight $AW_I^Z(J,S,N)$ is the product of share coefficients on the source specific price terms, $S_{I,S,J}^{MZ}$ and $S_{I,N,J}^{MZ}$, and the share of intermediate use of imports in industry J in total intermediate use in region Z, S_J^{MZ} :

$$AW_I^Z(J,S,N) = S_{I,S,J}^{MZ} \cdot S_{I,N,J}^{MZ} \cdot S_J^{MZ}$$

We now turn to the derivation the aggregate parameter, η_I^Z, occurring in equations (S20) and (S21) in Jomini *et al.* (1991). As in Section 3, we need only work with one of these equations:

$$x_{i,j}^{Dz} = q_i^z - \eta_i^z\left(p_{i,j}^{Dz} - p_{i,j}^z\right) \quad \text{(S20)}$$

Recall, $p_{i,j}^z$ is the share-weighted sum of $p_{i,j}^{Mz}$ and $p_{i,j}^{Dz}$.

Fix some aggregate I, J and Z. In levels values, domestic intermediate usage can be expressed as a price times a quantity:

$$D_{I,J}^Z = \left(DINT(I,J,Z) + TRD(I,J,Z)\right) = X_{I,J}^{DZ} \cdot P_{I,J}^{DZ} = \sum_{i \in I}\sum_{j \in J}\sum_{z \in Z} X_{i,j}^{Dz} \cdot P_{i,j}^{Dz} \quad (24)$$

In equation (25), we express equation (24) in differentiated per cent change form. We also implement our assumptions of maintaining functional form and that:

$$p_{i,j}^{Dz} = p_{I,J}^{Dz},\ p_{i,j}^{Mz} = p_{I,J}^{Mz} \text{ for all } i \in I, j \in J, z \in Z$$

and

$$q_j^z = q_J^Z \text{ for all } j \in J,\ z \in Z$$

to find:

$$\begin{aligned} & p_{I,J}^{DZ} + q_J^Z - \eta_I^Z \cdot S_{I,J}^{MZ}\left(p_{I,J}^{DZ} - p_{I,J}^{MZ}\right) \\ & = \sum_{i \in I}\sum_{j \in J}\sum_{z \in Z} W_{i,j}^{Dz}\left[p_{I,J}^{Dz} + q_J^Z - \eta_i^z \cdot S_{i,j}^{Mz}\left(p_{I,J}^{DZ} - p_{I,J}^{MZ}\right)\right] \end{aligned} \quad (25)$$

where

$$W_{i,j}^{Dz} = \frac{D_{i,j}^z}{D_{I,J}^Z}$$

Cancelling like terms and solving for η_I^Z in (25) we find:

$$\eta_I^Z = \sum_{i \in I}\sum_{j \in J}\sum_{z \in Z} \eta_i^z \cdot \frac{\left(\dfrac{D_{i,j}^z \cdot M_{i,j}^z}{D_{i,j}^z + M_{i,j}^z}\right)}{\left(\dfrac{D_{I,J}^Z \cdot M_{I,J}^Z}{D_{I,J}^Z + M_{I,J}^Z}\right)}$$

Appendix D contains further information on the parameters derived above and their TABLO code implementation.

Appendix D: Tables of correspondence

This appendix contains tables listing information about the SALTER parameters. The first table lists basic data to help identify the parameters, and gives the TABLO and textual correspondence. The second table lists the correspondence between the coefficients appearing in the aggregation formulae in the text and in the TABLO implementation.

Table D1: **Parameter concordance**

Header	*TABLO name*	*Formula*	*Description*
DP01	*ETA(I,Z)*	η_I^Z	Elasticity of substitution between domestic and imported *I* used in intermediate in region *Z*
DP02	*RHO(I,Z)*	ρ_I^Z	Elasticity of substitution between domestic and imported *I* used for investment in region *Z*
DP06	*BETA(I,Z)*	β_I^Z	Elasticity of substitution between domestic and imported *I* used for consumption in region *Z*
DP03	*GRHO(I,Z)*	β_{GI}^Z	Elasticity of substitution between domestic and imported *I* used by government in region *Z*
DP04	*SIGMA*(J,Z)	σ_J^Z	Elasticity of substitution between primary factors in industry *region* in region *Z*
RP04	*ETAI(I,Z)*	η_I^{MZ}	Elasticity of substitution between different sources of imported *I* used as an intermediate in region *Z*
RP02	*IETAI(I,Z)*	β_{KI}^{MZ}	Elasticity of substitution between different sources of imported *I* used for investment in region *Z*
RP01	*BETAI(I,Z)*	β_I^{MZ}	Elasticity of substitution between different sources of imported *I* used for consumption in region *Z*

Table D1: **Parameter concordance** (continued)

RP03	*GETAI(I,Z)*	β_{GI}^{MZ}	Elasticity of substitution between different sources of imported *I* for government use in region *Z*
DP05	*MBSHARE(I,Z)*		Marginal budget share of commodity *I* in region *Z*
DP13	*DL(Z)*	χ_I^Z	Elasticity of labour supply with respect to post-tax nominal wages in region *Z*
DP14	*H3(Z)*		Parameter for indexing wages to the CPI in region *Z*
DP15	*FRISCH(Z)*		Frisch parameter in region *Z*
DP17	*BT*		World-wide elasticity of substitution between freight sources

Table D2: **Concordance between text and TABLO formulae**

Parameter	*Comments*	*(Text formula) = (TABLO formula)*
η_I^Z	see Appendix C	$D_{i,j}^z = DI(i,j,z)$ $M_{i,j}^z = MI(i,j,z)$ $D_{I,J}^Z = ADI(I,J,Z)$ $M_{I,J}^Z = AMI(I,J,Z)$ $\left(\frac{D_{i,j}^z \cdot M_{i,j}^z}{D_{i,j}^z + M_{i,j}^z}\right) = WDM(i,j,z)$ $\frac{1}{\left(\frac{D_{I,J}^Z \cdot M_{I,J}^Z}{D_{I,J}^Z + M_{I,J}^Z}\right)} = AWDM(I,J,Z)$
ρ_I^Z	Substitute 'V' for 'C' in formulae of β_I^Z	
β_I^Z	see Section 3	$D_i^z = DC(i,z)$ $M_i^z = MC(i,z)$ $D_I^Z = NDC(I,Z)$ $M_I^Z = NMC(I,Z)$ $\frac{\frac{D_i^z \cdot M_i^z}{(D_i^z + M_i^z)}}{\frac{D_I^Z \cdot M_I^Z}{(D_I^Z + M_I^Z)}} = SC(I,i,Z,z)$

Table D2: **Concordance between text and TABLO formulae** (continued)

Parameter	*Comments*	*(Text formula) = (TABLO formula)*
β_{GI}^{MZ}	Substitute 'G' for 'C' in formulae of β_I^Z	
σ_J^Z	see Appendix B	$\frac{AFW_J^Z(m,k)}{S_{m,J}^Z}$ = *SAF(k,J,Z)*
		$AFW_J^Z(k,k)$ = *SAF(k,J,Z)*SAF(k,J,Z)*
		$FW_{j,z}^k$ = F(*k,j,z*)**IAF(k,J,Z)*
		$S_{m,j}^z$ = *SF(m,j,z)*
		$\frac{1}{S_{m,J}^Z - 1}$ = *IDSAF(m,J,Z)*
η_I^{MZ}	see Appendix C	$AW_I^Z(J,S,N)$ = *ASI(I,J,Z,S)*ASI(I,J,Z,N)*ASMJ(J,Z)*
		$\sum_{s \in S} W_{i,s,j}^{Mz}$ = *MINTAS(i,j,z,S)*IAI(I,J,Z,S)*
		$S_{i,N,j}^{Mz}$ = *SIAS(i,j,z,N)*
		$\frac{1}{S_{I,N,J}^{MZ}}$ = *IASI(I,J,Z,N)*
		$\frac{1}{S_{I,S,J}^{MZ} - 1}$ = *IASID(I,J,Z,S)*
β_{KI}^{MZ}	Substitute 'V' for 'C' in formulae of β_I^{MZ}	

Table D2: **Concordance between text and TABLO formulae** (continued)

Parameter	*Comments*	*(Text formula) = (TABLO formula)*
β_I^{MZ}	see Appendix A	
β_{GI}^{MZ}	Substitute 'G' for 'C' in formulae of β_I^{MZ}	
MBSHARE(I,Z)	see Section 3	
χ_I^Z	see Section 3	
*H*3(*Z*)	same as *DL*(*Z*)	
FRISCH(*Z*)	see Section 3	
BT	not aggregated	

Appendix E: Stored input files

This appendix contains copies of two stinp decks used in the aggregation facility. The first, **AGG.STI**, is used as an input to AGGMAP.FOR and is designed for use on a 37 commodity, 9 region version of the SALTER database. It creates an aggregation called TEST.MAP with 4 new industries and 2 new regions, in addition to the nonaggregated ones. To use this stinp deck, regional and industry specifications along with the output file name need to be changed appropriately. The second file, **ADDCES.STI,** is used to run the GEMPACK program MODHAR in batch mode. It was designed for use on the SALTER database aggregated down to 6 industries and 5 regions. It joins together the header array information from the files IO65.DAT and CES65.DAT into a new file AGG65.DAT. To use this stinp deck, you need only change the names of the input and output files.

AGG.STI

```
C          #CONTINUE WITH PROGRAM#
9          #NO. OF OLD REGIONS#
37         #NO. OF OLD INDUSTRIES#
Y          #Y=CORRECT ANSWER#
Y          #Y=YES,WANT TO AGGREGATE INDUSTRIES#
4          #NUMBER OF INDUSTRY AGGREGATIONS#
Y          #Y=CORRECT ANSWER#
2          #NO. OF INDUSTRIES IN AGG. NO.1#
1  2       #IND.'S IN AGG NO. 1#
Y          #Y=THAT RESPONSE IS CORRECT#
3          #NUMBER OF INDUSTRIES IN AGGREGATION NO.2#
4 5 6      #IND.'S IN AGG. NO.2#
Y          #Y=THAT RESPONSE IS CORRECT#
1          #NUMBER OF INDUSTRIES AN AGG. NO.3#
37         #THE IND. IN AGG.NO.3#
Y          #Y=THAT RESPONSE IS CORRECT#
3          #NUMBER OF INDUSTRIES IN AGG. NO. 4#
```

```
7 8 9       #IND.'S IN AGG. NO.4#
Y           #Y=THAT RESPONSE IS CORRECT#
Y           #THE ORIGINAL IND. SPECIFICATION IS OK#
Y           #WANT TO AGGREGATE REGIONS#
2           #NUMBER OF REGIONAL AGGREGATIONS#
Y           #Y=THAT RESPONSE IS CORRECT#
3           #NO. OF REGIONS IN AGG. NO.1#
1 2 4       #REGIONS IN AGG. NO.1#
Y           #Y=THAT RESPONSE IS CORRECT#
2           #NO. OF REGIONS IN AGG. NO.2#
3 5         #FIRST REGION IN AGG. NO.2#
Y           #Y=THAT RESPONSE IS CORRECT#
Y           #SATISFIED WITH REGIONAL SPECIFICATION#
TEST.MAP    #OUTPUT FILE NAME#
```

Now, the second stinp deck :

ADDCES.STI

```
F                       # REQUESTS FULL PROMPTS #
B                       # BOTH OUTPUT AND ERROR CONTROL #
Y                       # THIS IS BASED ON AN OLD HEADER ARRAY FILE #
IO65.DAT                # FILE NAME OF THE FILE CREATED IN AGGIO.TAB #
AGG65.DAT               # NEW FILENAME FOR THE COMBINED DATA FILE #
AW                      # ADD-WRITE THE NEW DATA TO THE CURRENT FILE #
H                       # NEW DATA IS FROM A HEADER ARRAY FILE #
CES65.DAT               # FILE NAME OF THE FILE CREATED IN AGGCES.TAB #
DP06                    # HEADER NAME TO ADD #
W                       # WRITE THIS ARRAY TO THE NEW FILE #
N                       # RETURN FOR A NEW PROMPT #
AW                      # ADD-WRITE THE NEW DATA TO THE CURRENT FILE #
```

```
H                   # NEW DATA IS FROM A HEADER ARRAY FILE #
CES65.DAT           # FILE NAME OF THE FILE CREATED IN AGGCES.TAB #
DP07                # HEADER NAME TO ADD #
W                   # WRITE THIS ARRAY TO THE NEW FILE #
N                   # RETURN FOR A NEW PROMPT #
AW                  # ADD-WRITE THE NEW DATA TO THE CURRENT FILE #
H                   # NEW DATA IS FROM A HEADER ARRAY FILE #
CES65.DAT           # FILE NAME OF THE FILE CREATED IN AGGCES.TAB #
DP08                # HEADER NAME TO ADD #
W                   # WRITE THIS ARRAY TO THE NEW FILE #
N                   # RETURN FOR A NEW PROMPT #
AW                  # ADD-WRITE THE NEW DATA TO THE CURRENT FILE #
H                   # NEW DATA IS FROM A HEADER ARRAY FILE #
CES65.DAT           # FILE NAME OF THE FILE CREATED IN AGGCES.TAB #
DP09                # HEADER NAME TO ADD #
W                   # WRITE THIS ARRAY TO THE NEW FILE #
N                   # RETURN FOR A NEW PROMPT #
EX                  # EXIT THE PROGRAM SAVING THE CURRENT FILE#
A                   # ADD ALL ARRAYS TO THE NEW FILE #
                    #A CARRIAGE RETURN#
                    #ANOTHER CARRIAGE RETURN#
ENTER YOUR NAME
00/00/92
ENTER THE HISTORY OF THE RUN
**END               # END OF HEADER FILE HISTORY INFORMATION #
Y                   # EVERYTHING IS OK, TERMINATE MODHAR #
```

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